Nonlinear Dynamics, Psychology, and Life Sciences, Vol. 10, No. 3, pp. 401-403.

Book Review

Computability Theory. By S. Barry Cooper. London: Chapman and Hall/CRC, 2004. ISBN 1-58488-237-9.

This is an unusual and in some ways, as the author says, very personal view of a fundamental topic that is often quoted and little understood. It contains both formal mathematics, starting from relatively elementary definitions, and also puts things in a social and historical context. Cooper delineates three stages in mathematical thinking about its foundations, based on Hilbert, Gödel and Turing, and then leads into computer fundamentals, and lastly chaotic dynamics. The author's sympathies and perspective are close to Turing's, and he covers computability from the starting point of Turing machines. The fact that Turing anticipated the theory of computers before the machines were in existence is one of the amazing things of our 20th century intellectual history.

In 1900 Hilbert posed 23 fundamental problems in mathematics, some of which are still unanswered. He asked, "Are there unsolvable problems in mathematics?" Do there exist computational tasks for which there is no valid program? Gödel's Incompleteness Theorem (1931) says in ordinary language "[A]ny consistent theory containing enough of the basic theorems of arithmetic is incapable of even proving its own consistency". In an age where absolute certainty was being replaced by doubt in physics, politics and morality, and which is still with us, the Incompleteness Theorem was not only destructive of much of Hilbert's program for mathematics, but can still be seized on by those who believe that truth with absolute certainty can only be known through revelation, and not through reason. We still have that conflict in the opposition of evolution and creationism. But unsurprisingly few that cite Gödel can have read, or could read the theorem. As one of the most readable reviews (Nagel & Newman, 1958, p.68) notes "Forty-six preliminary definitions, together with several important preliminary theorems, must be mastered before the main results are reached."

The Turing approach did not invalidate Gödel, but rather took an approach of postulating a reading head scanning an infinite tape running off in both directions, and showed that it could compute many but not all functions, by following simple recursive rules of transposition and replacement. This is the abstract form of machine language underlying user languages that we have today, such as Fortran, Pascal or C++. Cooper clearly sets out both the simpler Turing program and what is called an Oracle, with examples. The issue of importance for those wanting to simulate nonlinear dynamics on a computer is that there are two sorts of programs, P and NP. NP means ‘nondeterministic polynomial time decidable’, and P refers to “polynomial time computable” P defines a class of all the programs we commonly run on computers and excludes one for which the required time or storage expands impossibly.

None of these developments in any way make it impossible for us to do the sorts of calculations that appear in the methodology papers that are presented or reviewed in NDPLS, but they imply that some programs we might devise will never converge in our working lives to a stable limiting configuration, and others can produce astronomically large numbers that are useless. An example in Bayesian network analysis was created by Robinson (1977), where r(1) = 1, r(2) = 3, r(3) = 25, r(5) = 29281, and so on to r(10) = $4.2 \times 10^{18.}$ Another one of such functions in named after Goodstein (Cooper, p. 126) which mathematically terminates but in practice takes a virtual eternity.

Cooper does make some links to chaos theory, but oddly omits a seminal paper of Turing, that anticipated some fundamental work in what is now called synergetics (Tschacher, Schiepek. & Bruner, 1992). The idea of a bifurcation, as a change between modes of functioning of a dynamic system, under coupling of two attractors, was Turing’s (1952) explanation of morphogenesis, which is also called interactional bifurcation. We note that Turing moved to address this problem after his work on computability, and related work on indeterminacy not long before his tragic death in 1954.

Cooper’s excursion into chaos (p.379) is brief, and based on consideration of the Mandelbrot set, M. It can be shown that the complement of the set M is computably innumerable, but there is an unresolved dispute about M itself, the familiar and beautiful computer pictures are approximations and not the mathematical M itself.

For those whose appetite is stimulated by the book, but want to stay with real biological systems, the reader is guided to a paper by Cooper and Odifreddi (2003), and a good bibliography.

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