Nonlinear Dynamics, Psychology, and Life Sciences, Vol. 9, No. 3, July, 2005

Book Review

Unstable Singularities and Randomness, Their Importance in the Complexity of Physical, Biological and Social Sciences. By Joseph P. Zbilut. Amsterdam: Elsevier, 2004. 237 pages. ISBN 0-444-51613-1.

This book is presented as a relatively non-mathematical version of previous work (Zak,, Zbilut, & Meyers, 1997) intended to give the feel of some very important ideas to the more general reader. So if one wishes to check out some key ideas, that are repeatedly referenced and invoked but not formally defined, it is helpful to have access to other sources. It is replete with examples, some rigorous in physiology and others frankly speculative and vague, in art and society.

The key idea is to distinguish between deterministic chaos and stochastic chaos, the two have quite different properties as soon a system approaches a singularity, and Zbilut's main thesis is that deterministic chaos may be applicable in physics, but biological processes are only plausibly represented by stochastic chaos.

The notion of a singularity is strictly about mathematical models that at some place in their parameter space become utterly unspecified. In this context it is helpful to think of models of processes that run through time. If the model is deterministic, and it can be chaotic in the technical sense, then provided you have all the just recent data and the full set of rules for defining the model, then you can go forward or backward and say what happens next. When you land on a singularity then this predictive power locally temporarily vanishes; in a deterministic model you will be able to go onto a new path that again becomes predictable, the number and properties of paths is strictly known, and limited to two in the case of a bifurcation.

However, if the process is one of stochastic chaos, as Zbilut defines it, then when you land on a singularity the local prediction is never better than a random choice over a closed set of alternative future pathways that may be infinite in number. Zbilut's thesis is that biological systems have such singularities in their appropriate representations, and switch between being random and deterministic, in a

way that is not typical of idealised physical systems. He gives useful geometrical illustrations of the relevant mathematics.

As many of the tests that we can employ to identify deterministic chaos, such as Lyapunov exponents, have little meaning when stochastic indeterminacy is also present in a system, it is necessary to take a fresh look at data analysis. To emphasise the problem another way, the presence of singularities is associated with the presence of non-stationarity, and many tests of time series properties, such as Fourier spectra or Wavelets, are built on strong assumptions of stationarity; such assumptions are, given enough data, testably false.

A key idea, repeated many times, but somehow never formally defined, is a non-Lipschitz condition. This is about the absence of bounded derivatives at a point in an evolving process. A formal definition can be found in Hille (1959) or Nachbin (1981). The argument, which Zbilut helpfully illustrates with diagrams, is about what happens when a dynamic process hits on a singularity. At the singularity there is indeterminacy, the gradients are unbounded, so the Lipschitz condition is locally violated, they shoot off to infinity. In stochastic chaos what happens next is not a matter of landing on a new deterministic trajectory, as we get after a bifurcation, but instead there is a probability distribution (like a fuzzy cloud) of alternative pathways and no deterministic rule, at that moment, for deciding where to go next. So we have systems that are a lot of the time almost deterministic and hence modelled and predictable, and little intervening (probably unpredictable) epochs in which all is wild and random. Zbilut was not the first to use this mathematical property of non-deterministic equations to characterise processes in the behavioural sciences, Gregson (1993) gives examples in psychophysics that are much closer tied to data than the speculative discussion that Zbilut offers as soon as he strays away from the physiology of cardiac activity and analogous biophysical data., which are in his home area of competence. Cardiac activity is a central topic, partly because of the foundation work that Winfree (1987) laid, and which the beginning reader would find illuminating to read first in order to see the advances that Zbilut has now added to our understanding. He has in a wider sense drawn our attention to new conceptual tools that have to be kept in mind as alternative explanations for the evolution in time of unruly data, and hopefully got us past the sort of chaos that might only exist in a few purely physical stationary systems.

What Zbilut suggests augments our previous knowledge that cardiac activity is not a regular rhythm (a mistake often made by clinicians) but exhibits the long-range correlations of highly complex

nonlinear processes (Peng et al, 1993) and also has transient episodes whose dynamics are stochastic and have some survival value. The heart tries to avoid getting stuck on its own intrinsic singularities.

The excursion into artistic preferences would probably have been better expunged, Zbilut observes correctly that artistic choices vary over time, can change abruptly, and where they go to next can be unpredictable. But it does not follow necessarily that they are likely candidates for being modelled by stochastic chaos; we need more than just what the physicists call phenomenological data.

This jumping between deterministic segments was expressed far more stylistically by William James, in his *Principles* (I, 243) where he wrote that our mental life "like a bird's life, seems to be made of an alternation of flight and perchings". Our mathematics have got better but our prose imagery has degenerated.

Unfortunately, the book is replete with copyediting faults; sentences are ungrammatical, symbols are missing from equations, even one graph has no lines in it. This does not matter if you know the subject already, but for the beginner it is disheartening. We hope Zbilut does better next time, as the book is expensive.

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