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Book Review

Chaos and Time-Series Analysis. By Julien Clinton Sprott. Oxford University Press, Oxford, UK, & New York, USA. 2003. xx + 507 pp; 298 figures. ISBN 0 19 850839 5 (hardback), 0 19850840 9 (paperback).

This book is the most comprehensive and clear text for learning dynamics and many related topics of fractals and complexity that I have encountered. It is appropriate for both classroom use, and for scientists working in a wide variety of fields. The emphasis is on concepts and applications rather than proofs and derivations. It is 'hands-on', with practical exercises and a programming project in each chapter. Dynamics programs, such as *Berkeley Madonna* (Macey, Oster, & Zahnley, 2000) or those by Sprott and Rowlands (1995) are most useful for additional explorations.

There are three appendices. The first is a catalog of common chaotic systems, each with a graph, equations, typical parametric values and initial conditions, Lyapunov exponents, Kaplan-Yorke dimension, correlation dimension, and a major reference. The second appendix gives useful mathematical formulas. The third is a list of relevant journals. There are 715 bibliographic entries and 298 figures. There is an excellent support website with color versions of many of the figures and much supplementary and updated information (including answers to some of the exercises) at http://sprott.physics.wisc.edu/chaostsa/.

The *Introduction* (Chapter 1) mentions several examples of chaotic astronomical, physical, and nonphysical systems, many with photographs and diagrams. There is also a section on electrical circuits, which are of value whether one is intrinsically interested in electronic circuits or not. Some circuits can be easily and inexpensively constructed. A nice aspect of the book is the inclusion of brief footnotes on individuals. The programming project concerns the logistic equation. The first exercise to derive a set of four first-order ordinary differential equations for the three-body problem is one on which many have spent much time since Poincaré first stumbled over it. I found myself trying to cheat already. Sprott's website contained a link to Wolfram's website that told more about the three-body problem.

Programming the logistic equation prepares you for the next chapter on *one dimensional maps* (Chapter 2) where the logistic equation is the main object of study, and where the programming exercise is continued with an exploration of its bifurcation diagram.

Nonchaotic multi-dimensional flows (Chapter 3) moves on to the continuous, multivariate case where time moves continuously rather than in discrete steps. It starts with a simple first-order, explicit, linear differential equation for population growth and decay. He uses this model to provide the distinction between maps and flows, which he summarizes in a table. This comparison is continued with some multi-dimensional models including the simple harmonic oscillator (to be taken up again in Chapter 8 on Hamiltonian systems), the driven harmonic oscillator (from the introductory chapter), the van der Pol equation, and others. Under the driven harmonic oscillator, he takes up an important topic, namely that of converting a system of nonautonomous differential equations into an autonomous system, making it amenable to solution for representation in state space. The chapter finishes with an explanation of the various numerical methods of solving equations, the Euler, Leap-frog, and Runge-Kutte second-order and fourth-order methods.

Dynamical systems theory (Chapter 4) is the heart of the book. Saying that it covers two-dimensional equilibria, stability, the damped harmonic oscillator (again), saddle points, area contraction and expansion, nonchaotic three-dimensional attractors, chaotic dissipative flows (some 19 are summarized in a table), would hardly reveal the importance of these concepts. It finishes with 'shadowing' on the relationship between a true trajectory and the computed one. Eigenvalues (characteristic exponents and multipliers) and the use of the determinant of the Jacobian are clearly developed. You need never have heard of these before.

Lyapunov exponents (Chapter 5) are important for depicting the converging and diverging properties of a chaotic attractor. These are closely related to the eigenvalues upon which they depend but important differences are noted and are summarized in a table. There are important distinctions and relationships between local and global Lyapunov exponents. Although related, the Lyapunov exponents measure behavior of a system over time, while the *Kaplan-Yorke (Lyapunov) dimension* is a measure of the complexity of an attractor. It sets the lower bound of the number of variables required to model the dynamics. Since a goal of modelling is to try to minimize the number required, establishing that minimum is important.

Strange attractors (Chapter 6) begins with 12 properties such as limit set, invariance, stability, sensitivity to nearby initial conditions, and yes, aesthetics. He develops the idea of the probability of chaos with a dynamical system (the proportion of parameter space yielding chaos) and search techniques for finding values yielding chaos. Sprott is well known for developing this approach. He is also known for programs that display such attractors in 3D.

I consider *Bifurcations* (Chapter 7) the most important property of nonlinear systems. Sprott considers them important "because they provide strong evidence of determinism in otherwise seemingly random systems, especially if the parameters can be repeatedly changed back and forth across the" bifurcation point (p. 159). I consider them important from a slightly different perspective, that of a system that can explain differing patterns of experimental data whose connection may not have been previously noticed and for which different models might have been suggested. Both perspectives represent the same parsimonious point of view. In addition to many familiar bifurcations, it also includes homoclinic and heteroclinic bifurcations, and the examination of Lyapunov spectra as a function of control parameters especially with bifurcations that involve *transient chaos* and *crises*.

Conservative systems include familiar examples like ideal frictionless pendula for which Hamiltonian equations may be used (*Hamiltonian chaos*, Chapter 8). It also includes simplectic maps, which are important for systems where "the numerical methods may not precisely conserve the invariants" (p. 199). It also explains the mathematical relationship between the flow (in n dimensions) and the map (in n-1 dimensions) to approximate its Poincaré section conservatively.

Since dynamical systems evolve in time, *Time-series properties* (Chapter 9) are the heart of the analyses of data obtained from them. Sprott provides a quick review of traditional linear methods (including topics of stationarity, detrending, noise, autocorrelation, and Fourier analysis). Comparison of a noise signal with one produced by a one-dimensional map illustrates the use of surrogate data (Monte Carlo methods) and return maps to determine the extent to which the data contain deterministic as well as stochastic information. The final part of the chapter introduces time-delay embeddings used for attractor reconstruction and for determining the dimension of an attractor.

"One of the most important applications of time-series analysis is" *Nonlinear prediction and noise reduction* (Chapter 10, p. 243). Comparison of models with data lies at the heart of both prediction and evaluation of models. There are limitations of the linear methods of autoregression and some practical limitations of nonlinear methods, but a method called *random analog prediction* works rather well. Similarly, linear noise reduction techniques are of little use with chaotic data, and thus *state-space averaging* is preferred. Prediction depends on evaluating divergence of nearby trajectories, which, for data not modeled by equations, means measuring the rate of divergence and summarizing them in Lyapunov exponents. Several methods of doing this are compared.

The subject of embedding from the previous chapter is examined further using *false nearest neighbors*, a method for estimating the optimal Cartesian embedding dimension by systematically increasing the embedding dimension until separation of nearby points no longer occurs. While traditionally the main use has been to get a best view of the attractor, to help determine when other measures of fractal dimensionality have saturated (become asymptotic), and to estimate the likely number of variables involved in the dynamical system under investigation, Sprott points out other uses, such as evaluating if sufficient points are in a neighborhood to support prediction. (Stewart has described the extension of the technique to multivariate data; see Abraham, 1997). Other methods include *recurrence plots* and *space-time plots*, for which Sprott evaluates various computational algorithms for simplicity and efficiency.

Principal component analysis is useful for noise reduction, in estimating dimension, and in building model equations with polynomials. Among *artificial neural network predictors*, *single-layer feed-forward networks* are mentioned for their computational and conceptual ease and for the large literature on optimization and training. Two methods are mentioned: *multi-dimensional Newton-Raphson* and a simplified variant, *simulated annealing*. Chapter 10 covered a lot of topics in a rather short space, but nonetheless, provides a great introduction to them.

Fractals (Chapter 11) takes a broader view than simply a "geometric manifestation of chaotic dynamics". Examples include Cantor sets, fractal curves (devil's staircase, Hilbert curve, Koch snowflake, the basin boundary of a Julia set, and the Weirstrass function, fractal trees, fractal gaskets, fractal sponges, random fractals, and fractal landscapes (forgeries). A consideration of fractal properties in nature completes the chapter.

A few of the methods are presented for the *Calculation of the fractal dimension* (Chapter 12). These include the *similarity dimension*, the *capacity dimension*, and the *correlation dimension*. *Kolmogorov-Sinai entropy*, a function of Lyapunov exponents measures the loss of information in forecasting. The *BDS statistic* measures the amount of determinism in a time-series by evaluating its departure from randomness. It depends on the correlation dimension. *Minimum mutual information* is a measure that is used to help estimate lags for a time-delay embedding. It thus plays a role in the determination of the embedding dimension and attractor reconstruction. Practical considerations include the speed of calculations, requirements of size of data sets, precision, noise, the use of multivariate data, filtering, missing data, sample spacing, and nonstationarity. The advantages of using multivariate data were well stated, but a more extended treatment might help to underline its importance. The final method is that of computing the fractal dimension of graphic images.

Fractal measure and multifractals (Chapter 13) may help with nonhomogeneous fractals or when the computation of the correlation dimension doesn't converge nicely. This process involves an extension of the fractal dimensions into a *spectrum of generalized dimension.* Alternative characterization of multifractals can be achieved with the *similarity spectrum* or a *dynamical spectrum of entropies*. This is a complex but highly enlightening chapter.

Nonchaotic fractal sets (Chapter 14) is a discussion of fractal objects generated by systems other than chaotic dynamical systems. These include *iterated function systems* (the chaos game and affine transformations), which can be used to create images simulating natural objects and to compress images. They can also be used to create patterns from data which give visual clues as to deterministic and stochastic features of the data—the IFS clumpiness test.

Spatiotemporal chaos and complexity (Chapter 15) deals with chaos exhibited over spatial as well as temporal dimensions. Complexity refers to broad class of subjects not unified by any theory, but many of them depend on dynamical system concepts. The term includes not only chaos, fractals, and neural networks, and artificial life, but also complex dynamical systems which in turn includes cellular automata, lattices, and self-organization. Some special systems that have had widespread deployment in many sciences include *self-organized criticality*, the *Ising* model, *percolation*, the last being of interest as a *complex adaptive system*, *coupled lattices* and *infinite dimensional systems.* A summary of spatiotemporal models in terms of discrete or continuous spatial, temporal, or state conditions is given, along with consideration of criteria and trade-offs for usage.

His final concluding remarks are of special interest as they deal with issues of free will and responsibilities, social, ecological, and aesthetical.

This book is an indispensable addition to my bookshelf. It provides a good foundation in almost all aspects of dynamical systems theory. For those with the minimum recommended mathematical background, the explanations and support material will fill in the necessary updating of your mathematical knowledge. The explanations were clear, and covered almost all related aspects of each subject. The later chapters sometimes tried to compress many topics into them so that the compression may benefit from some supplementation. The extensive and evolving website back-up makes the book unique and even more valuable. The book is thus perfect for self-instruction, or for use as a classroom textbook, and as a reference work for those in any field of science.

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