Nonlinear Dynamics, Psychology, and Life Sciences, Vol. 7, No. 3, July 2003 (© 2003)

Book Review

Nonlinear Dynamics and Chaos: Where Do We Go From Here? Edited by S. J. Hogan, A. R. Champneys, B. Krauskopf, M. di Bernardo, R. E. Wilson, H. M. Osinga, and M. E. Homer. Institute of Physics Publishing, Bristol, 2002, 358 p. ISBN 0-7503-0862-1.

This timely and important book is a record of papers presented at a conference in Bristol and is very well edited, and produced, though it lacks a subject index. It includes contributions from Britain, Germany, France, Japan, Spain, and the USA. As the preface tell us, it was born out of a suspicion that the theory and practice of dynamical systems had reached a plateau, and a wish to assess what still needs to be learnt. The message is clear; "if dynamical systems theory is to make a significant long-term impact, it needs to get smart, because most systems are ill-defined through either stochasticity, delay, spatial extent and inhomogeneity or the finite-time nature of real world data." Anyone who has worked in the areas of interest to SCTPLS can recognize these problems, and the sense of frustration they can engender, immediately. It is still sad (and a consequence of the irrationalities in the sociology of the sciences) that the authors of these papers are apparently sometimes unaware of any work outside the specific physics and physiology problems that properly motivate their activities, even when that work might cast more light on the difficulties they realistically identify.

The papers are fortunately not restricted to physics but include biological problems, and indeed perhaps the most striking is the revision of the already classic work by Bressloff and Cowan on geometric visual hallucinations which had appeared in *Philosophical Transactions of the Royal Society, B*, in 2001. Two of the three areas selected for coverage were neural and biological systems, and spatially extended systems, but obviously the themes can overlap in any real case.

The first paper, by John Guckenheimer, on bifurcation and degenerate decomposition in multiple time scale dynamical systems, serves as a framework for much of what follows. Contrast this with the coverage that a decade ago was essential basics, as provided by Martelli (1992), or with a contemporary parallel treatment by Nishiura (2002) and one gets an appreciation of the breadth to which dynamical theory has expanded. Guckenheimer expands theory to only two parallel time scales, one fast and the other slow, but even here the situation is still fuzzy. The well-worn approach of starting from the Logistic equation, or the Hénon or Lorenz systems will not do, in part for computational reasons and in part due to the extreme disparity and instability of scales in the phase spaces. The exploration of a model that can alternately evolve in slow or fast time scales is masterly in its subtlety, this still unfinished work reveals at least six sorts of degeneracies in the evolution of orbits.

The reader who has not yet heard about canards will benefit from studying this chapter.

We may here pass over the work on quantum mechanics, but the treatment of high dimensional chaos by Uwe an der Heiden, in Chapter 3, though strictly mathematical, addresses a diversity of systems including mixed feedback and chaos, and the solution of retarded difference equations. This sharpens up our understanding of the various types of difference equations with continuous arguments and maps on the interval where

$$x_n = f(x_{n-1})$$

where n is an integer can have a fractal attractor with dimension equal to one, whereas

$$x(t) = f(x(t-\tau)) \qquad t0,$$

τ takes continuous values, can have a strange strange attractor with dimension infinity!

Christopher Jones in Chapter 4 discusses creating stability out of instability. The area of application is nonlinear optical fibres, but the questions he raises are general and demand explicit quotation: "The strength of dynamical systems lies in its adaptability, but much of the adaptation has not yet taken place," and "Is the main distinction between *linear* and *nonlinear* or between *local* and *global*? ... The fascination with the phenomena exposed by dynamical systems, such as chaotic motion, for the applied scientist has often been characterised as nonlinear ... However, this is misleading. The driving force of chaos, for instance, is the elementary linear effect of exponential stretching near a saddle point. Nonlinearity merely supplies the recurrence needed to have the dynamics repeatedly experience the saddle effect."

Chapter 5 by Van Wiggeren, Garcia-Ojalvo and Roy is concerned with a nonlinear approach to spatio-temporal communication. Though it is a strictly engineering approach we immediately learn that it has relevance to the intraspecies communications of cuttlefish. Species other than humans use wavelengths that are not readily accessible to us, and optical fibre transmissions may be more revealing in understanding analogously what happens. Particularly useful is the reference (p. 112) to the work on spatiotemporal communication with synchronized optical chaos, where a static image has a real part that is encoded as chaotic, and decoded back to the image and thus recovered by the receiver. As the authors observe, we are nowhere yet near reaching the limits of communication theoretically possible with electromagnetic radiation. "It is clear that biological systems process vast amounts of information more efficiently and more flexibly than we have learnt to do so far."

Edgar Knobloch reviews outstanding problems in the theory of pattern formation in Chapter 6, these are mostly in chemical media, yet he notes but tantalizingly does not take up the point that pattern-forming systems behave almost like a colony of living organisms. He employs weakly nonlinear theory, there seems to be a parallel to Wolfram on cellular automata here, but that gets no mention in the 162 references cited. Actually I suspect there is a matter of importance here for social psychologists, because on page 158 the "model equations whose structure depends sensitively on the assumptions made about the spatial and temporal scales in the system, both intrinsically and extrinsically imposed" cannot be usefully derived from traditionally equivalent theory. This is because traditional theory "assumes that all interactions are local, local in both space and time, and in phase space."

Dropping those assumptions leads to very heavy mathematics, of which examples are given. The mathematician's task is then to look for simplifications without loss of rigour.

We may be excused for passing over fluid dynamics (in Chapter 7) but time-reversed acoustics and chaos surveyed by Mathias Fink was a fascinating surprise. Fink develops the theory of time-reversal mirrors which reflect scalar waves. The theory extends to reversals in chaotic cavities. There has been evidence about some reflection of EEG-like waves inside the human skull, but these are not mentioned.

The last three chapters are of particular interest to the life sciences, as vision, cardiac activity, and developmental perspectives are considered. Bressloff and Cowan describe both real data and simulations of spontaneous pattern formation in primary visual cortex. I found their summary of mapping transformations from eye to V1 of the cortex the most readable I have seen for some time, enabling the reader who is not a psychophysiologist to get a feeling for what happens in feature and connectivity processing. The uncritical acceptance of the much-cited work of Hubel and Wiesel comes in for some cautious reevaluation (p. 286) Cells in the visual cortex show preferential responses to features such as orientation, left/right dominance, and direction of movement. But the topography changes continuously except at singularities, and periodicity in cell structure. So there is both local and long-range connectivity and the network can carry information about global structure as well as about details of input patterns. The link to dynamics is made via what is called the Turing mechanism, which Turing introduced as a means of considering the formation of patterns in animal coat markings; the idea leads into what is called diffusion-driven instability, and it was extended into neural network theory by Wilson and Cowan. By more deep mathematics the Turing formulation was extended by Ermentrout and Cowan to describe the stability and amplitude of activation patterns. A final step in the argument was added by von der Malsburg who showed that pattern formation can occur in a neural network provided that competition is present The link to the generation of visual hallucinations is (p. 281) "based on the idea that some disturbance such as a drug or flickering light can destabilize. . . . inducing a spontaneous pattern of cortical activity that reflects the underlying architecture (of the visual cortex)." It is then possible strikingly to match the reported forms of hallucinatory forms induced by LSD or marihuana, or in primitive art, with some computer generated visual field images. This includes matching oscillatory activity, and studying both driven and spontaneously generated patterns.

Ermentrout and Osan (Chapter 12) complement the previous chapter with a review of how the visual system develops: "The precise structure of the map is different in every animal so that it is not 'hardwired' genetically but rather arises during development." The older ideas of Hebb about plasticity in neural development get refreshed. Optimistically this chapter ends with the remark that (p. 345) "Our approach is equivalent to the law of mass action and constraints are built in, so that there is never any problem with unbounded negative solutions. By assuming correlated decay of weights as well as growth, we are able to obtain pattern formation even if the cortical weights are positive and the correlations between features are non-negative." Interestingly in this context the work of Grossberg, which heavily employed adjacent regions of activating and inhibitory neurons, is not quoted.

The final brief chapter is by William Ditto on "Spatio-temporal nonlinear dynamics: a new beginning." I cannot do better than quote him (p. 349): "While the study of low-dimensional dynamical systems has been successful, there are serious theoretical and practical problems in extending those studies towards understanding dynamical systems that are noisy, non-stationary, inhomogeneous and spatio-temporal. In my view the key issue is the *interplay between temporal and spatial dynamics*."

He notes that when we leave isolated complex systems and look at coupled systems or ones that evolve then we enter a world that is so far analytically intractable. Two examples are used, heart fibrillation, and the effects of dynamical clamping on leech neurons. Both create unsolved problems for the theoretician even as a partial description is achieved. A crucial observation here is that the systems we aim to model in the life sciences are ones that live in continuous real-time feedback and under adaptive behavioral control.

Perhaps we should start to demand that work submitted for publication should always include some awareness of what adding feedback loops to an apparently closed though dissipative nonlinear model would do. Unfortunately there are not yet many of us who could meet that standard. The very richness of this book, in both theory and real-world applications, makes it difficult to summarize and even more difficult to put down. It reveals what a lot of catching up some workers in psychology need to do, and how the future demands more hard work in both computation and rigorous model structuring, and less endless reiteration of the old favourite examples, that have still not been shown to be precisely the models that our data are crying out to have considered, even as qualitative metaphors. I recommend that one tries to grasp the very first model by Guckenheimer on pages 4–10 to get a feel of what we are up against in order to make progress.

REFERENCES

Martelli, M. (1992). *Discrete dynamical systems and chaos.* London: Longman Scientific and Technical.

Nishiura, Y. (2002). Far-from-Equilibrium Dynamics. [Translations of Mathematical Monographs, vol. 209] Providence, Rhode Island: American Mathematical Society.

Robert A. M. Gregson
Australian National University
Canberra, ACT Australia
E-mail: ramgdd@bigpond.com