Nonlinear Dynamics, Psychology, and Life Sciences, Vol. 1, No. 3, 1997

Book Review

Understanding nonlinear dynamics. By Daniel Kaplan and Leon Glass. Textbooks in Mathematical Sciences Series. New York: Springer-Verlag. 1995.

The attainment of global literacy has been a goal of educators since the early days of this century. Although this goal has not yet been achieved, a significant proportion of the population has been given the ability to read, write and converse with sufficient fluency to make effective use of modern media in their daily lives. Many citizens enjoy fluency in a second or third language. Sadly however, only a tiny minority of the populace has achieved any fluency in that mode of discourse which has been one of the towering achievements of modern intellectual thought: mathematics.

For the majority of people, mathematics is considered synonymous with arithmetic. The great revolution of the 'new math' of the sixties fizzled, leaving few people convinced of the value of mathematics in their working lives. The problems studied were often far removed from direct experience, apart from accounting problems which, while useful, only further reinforced the link with arithmetic. Worse, mathematics has been taught not as a mode of thought, as a rich and powerful language, but instead as an arcane and inflexible set of rules for the manipulation of arbitrary symbols, devoid of relevance.

Although mathematics has flourished in the sciences, its greatest impact has been in physics, computer science and cryptography. The use of mathematics has languished in the social sciences. The reasons for this are complex. Blame must be shared by mathematicians and social scientists alike. Mathematics departments have tended to be arrogant and insensitive to the needs of social science students. The examples and problems presented to students have tended to be drawn from physics or pure mathematics, holding little relevance for future social scientists. Till recently, few research mathematicians expressed much interest in the study of social, indeed even biological, problems. Social science departments on the other hand consistently leave their students at a disadvantage by not requiring sufficient mathematical fluency to enable them to pursue even entry level

1090-0578/97/0700-0233$12.50/0

mathematics courses. They ignore mathematical approaches to their subject matter and tend to avoid having mathematically literate faculty to act as role models.

The existence of these 'two solitudes' was understandable at a time when mathematical approaches had little to offer for understanding complex social processes. In the past three decades however there has been a slow but persistent revolution taking place in mathematical thought. The rediscovery of chaotic dynamics in the 1960's opened the door on a set of ideas and tools which are ideally suited to the study of complex systems. Paralleling the development of chaos theory has been that of complex systems theory itself, originating in solid state physics and venturing forward into cellular automata and neural networks and giving rise to ideas such as self organization, emergence, self organized criticality, the edge of chaos. Their powerful new tools and ideas offer hope that complex social processes will eventually yield to formal analysis, transforming our understanding of them.

In order for such a revolution to occur in the social sciences, it is first necessary that a sufficiently large cohort of mathematically literate researchers be produced in order to carry the initiative forward and to sustain it through the often stormy period of paradigm emergence. Until recently, there were few textbooks on these subjects which would be accessible to the typical social scientist. Several superb textbooks exist, but are slanted either to a seasoned mathematical audience (Devaney, 1989; Wiggins, 1990) or to a physics audience (Gutzwiller, 1990; Lichtenberg & Lieberman, 1992) and follow the traditional theorem-proof style of teaching.

Now comes Kaplan and Glass. In *Understanding Nonlinear Dynamics*, they have struck a balance between rigor and interest, between the achievement of technical competency and the understanding of ideas. The text is accessible to anyone with a background in first year mathematics, primarily calculus and linear algebra and a willingness to work. The book is without proofs, focussing instead on motivating and illustrating the basic ideas of dynamical systems. That does not mean a lack of rigor. Far from it. Each chapter has been carefully planned and is free of factual errors. Having eschewed proof in favor of geometrical arguments, they have made a textbook which should be within the reach of a very broad audience in both the sciences and social sciences.

It is not for the faint of heart, however. Students with a high level of math anxiety will still require careful tutoring since the material is sophisticated. But it is motivated with a broad selection of interesting and relevant examples, set aside in sections aptly named 'Dynamics in Action'. These examples include: chaos in periodically stimulated heart cells, locomotion in salamanders, spiral waves in chemistry and biology, traffic on the internet, metastasis of malignant tumors, action potentials in nerve cells, fluctuations in marine populations, predicting the next ice age.

The book proceeds at a leisurely pace beginning with the study of finite difference equations. Here they introduce the basic concepts of dynamics: fixed points, cycles, quasiperiodicity, chaos emphasizing geometrical methods. Chapters 2 and 3 introduce the students to cellular automata, boolean networks and fractals. In Chapters 4 and 5 they explore the dynamics of one and two dimensional systems in detail, emphasizing the detection of fixed points, cycles and stability analysis. Again they provide the student with a solid foundation in the basic tools of analysis using mostly simple geometric and algebraic arguments. They even manage to introduce the Poincaré Index theorem which is usually relegated to senior undergraduate or graduate courses. Finally they end the book with a chapter on time series analysis.

As Kaplan is a pioneer in the study of the detection of determinism in time series data, this chapter offers up to date and cutting edge material. This chapter alone is worth the price of the book, describing the ideas of time series analysis in a clear and cogent manner. When coupled with the exercises, particularly the one on the construction and use of surrogate data methods, the student comes away with a solid basis for further pursuit of these important new tools of data analysis.

There are 99 exercises and 26 computer problems. The exercises vary from trivial to difficult. The range is sufficient so that problem sets can be tailored to virtually any level of competence within the class. Kaplan and Glass used their text primarily for third year biology students. I have used it as an introductory text for graduate psychology students. My students found the text to be readable, informative and the problems challenging but not overwhelming. One must be prepared to provide lots of worked examples in order to demonstrate the ideas and to help the students achieve comfort in and mastery of the material. The math phobic will still find the material overwhelming unless given special attention. I would strongly suggest using a tutorial group in addition to lectures unless the students are unusually mature mathematically.

If I have any quarrel with the authors it is with the rather sparse attention given to the study of chaotic systems. Although chaos is introduced in chapter 1 in the context of finite difference equations, it is virtually ignored in the remainder of the book. In total only 10 pages are devoted to chaos. The authors have chosen to focus on one and two dimensional continuous systems, clearly for sound pedagogical reasons given the ease of use of geometrical visualization to aid in the solution of problems. But by restricting themselves to such low dimensional systems they must abandon any serious survey of chaos. I think the text would have been well served by even a small chapter on three dimensional systems providing an introduction to the concepts of the strange attractor, homoclinic points and tangles, stable and unstable manifolds. These could have been handled in a purely visual way as was done by Abraham and Shaw (1984).

Quibbles aside, I think that this is a fine introductory textbook and its widespread use will help to improve the awareness and competence of biologists and social scientists in the use of mathematical models and modern methods of time series analysis. It would also serve as a useful text for established researchers interested in learning what these new approaches have to offer. I would highly recommend it for every social scientist's bookshelf.

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