Book Review

Coping with chaos: Analysis of chaotic data and the exploitation of chaotic systems. Edited by Edward Ott, Tim Sauer, and James A. Yorke. New York: Wiley. 1994. 418 pages.

This book is important for several reasons. First of all, it uses the word "chaos" in the title on a book that is expectionally competent and sophisticated. This should help legitimize the word when many of us have been concerned that its popularization may have destroyed its academic usefulness. We seem to want both the respectability of disciplinary academics and the metaphoric creativity that transcends the usual academic market place. This book well affirms the former role of dynamics as an important scientific strategy.

Second, while the development of theoretical dynamical models is considerably advanced, the design and analysis of experiments in our fields has been insufficiently developed (Abraham, 1997). This book and the classic review paper by Abarbanel, Brown, Sidorowich, and Tsimring (1993; now expanded into a fine book: Abarbanel, 1996) provide some very sophisticated techniques that should do much to help develop this embryonic field.

Third, many of us in the psychological, social, and life sciences have a mature mathematical conceptual outlook but not the time or training to be really proficient in mathematics to the extent we might wish. Many of us need mathematical colleagues for some assistance with our efforts in dynamics. This book can help us considerably. Although this book's mathematics may be a stretch, its exposition is so clear that we can follow the basic concepts and improve our mathematics at the same time.

This book has three main parts, 15 chapters, and reprints of 42 classic papers. Part I, "Background," is a clear and concise primer on some basics of dynamics (pp. 2-62). The first chapter is a review of dynamical systems, attractors, and chaos. This 12-page chapter gives an excellent summary of the basics, and clear distinctions between an invertible map (logistic) and a noninvertible map, between attractors from conservative Hamiltonian systems obeying Liouville's Theorem and attractors from nonconservative systems (this distinction involves evaluating volume change in an attractor

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using a Jacobian determinant). The box-counting definition of fractal dimensions is given, and then a definition of chaos (it specifies sensitive dependence on initial conditions; debated by some) using Moon and Holmes' (1979) beam apparatus as an example. It also shows how the second-order nonautonomous equation of Duffing can be converted into autonomous first order differential equations, and ends by providing a definition of Lyapunov exponents. I mention these as examples of terms that can seem a bit formidable at first but become crystal clear in their use by these authors. The book may provide motivation for review and further learning.

The remaining four chapters of this introductory primer focus on issues of measuring properties of chaotic attractors: dimension, symbolic dynamics, Lyapunov exponents and entropy, and the theory of embedding.

Part II, "Analysis of Data from Chaotic Systems" is comprised of four chapters of 3-6 reprints each (pp. 65-203). Each chapter starts with a review of basic principles by the editors and with a description of what each paper contributes to the subject. These papers include the great classics that we have all seen quoted over and over. Few of us have had the convenience of having them on our shelves, an opportunity provided by this book.

For example, in chapter 6, *The Practice of Embedding*, the paper by Packard, Crutchfield, Farmer, and Shaw (1980) 'Geometry From a Time Series', is the foremost introduction to the time-lagged (delay-coordinate) technique of reconstructing an attractor from a single time series. Broomhead and King's (1986) 'Extracting Qualitative Dynamics from Experimental Data', added filtering using singular value decomposition to the technique. Kennel, Brown, and Abarbanel's (1992) 'Determining Embedding Dimension for Phase-space Reconstruction Using a Geometrical Construction', added the false-neighbors improvements to diminish the number of dimensions needed for emedding and assisted in distinguishing deterministic from stochastic components. Kaplan and Glass' (1992) 'Direct Test for Determinism in a Time Series' adds another technique for evaluating deterministic and stochastic contributions by comparing an averaged vectorfield with that from a randomized vectorfield. I liked the editors pointing out that these techniques, as they also did in their Chapter 5 (from Part I) on the theory of embedding, were appropriate for simultaneous multiple variables as well as time-delayed variables (Abraham, 1997; Sauer, Yorke, & Casdagli, 1991; Stewart, 1996).

Chapter 7, *Dimension Calculations*, includes: Albano, Muench, Schwartz, Mees, and Rapp's (1988) 'Singular-value Decomposition and the Grassberger-Procaccia Algorithm'. It provides results of the Broomhead and King procedure for measuring the correlation dimension. Eckmann and Ruelle's (1992) 'Fundamental limitations for estimating dimensions and Lyapunov exponents in dynamical systems' and Ding, Grebogi, Ott, Sauer, and Yorke's (1993) 'Plateau Onset for Correlation Dimension: When Does it Occur?' both address issues of inadequate data. Theiler, Eubank, Longtin, Galdrakian, and Farmer's (1992) 'Testing for Nonlinearity in Time Series: The Method of Surrogate Data' is known to our field because they used EEG among their test time series and so it is frequently quoted in our disciplines. Econometric and epidemiological data have also been so tested. They introduced Monte Carlo techniques for establishing a null hypothesis and statistical testing of attractor invariants. The Brandstater and Swinney (1987) 'Strange Attractors in Weakly Turbulent Couette-Taylor Flow' examined portraits, power spectra, circle maps, and dimension estimates to evaluate bifurcations and problems of data requirements. The final reprint of this chapter, Guckenheimer and Buzyna's (1983) 'Dimension Measurements for Geostrophic Turbulence' is also a study of bifurcation in turbulence. Their study used multiple simultaneous time series. The editors note, "It is our feeling that simultaneous measurements will often give superior results [to the delay-coordinate procedure], and should be used, if available" (p. 106).

Chapter 8, *Calculation of Lyapunov Exponents*, contains three papers which take up the evolution of these computations since Wolf, Swift, Swinney, and Vastano's (1985) heuristic method for estimating the largest exponents. The idea is to convolute an m-frame of orthogonal vectors over a trajectory to get at local linear dynamics. The first paper here, Eckmann, Kamphorst, Ruelle, & Ciliberto's (1986) 'Liapunov Exponents from Time Series' is an application of this method to synthetic and experimental data. It is one of the first to provide computation of several characteristic exponents. The next paper, Bryant, Brown, & Abarbanel's (1990) 'Lyapunov exponents from observed time series' adds the use of hypothetical higher degree polynomial models during the convolution. See Brown, Bryant, and Abarbanel (1991) for more details of this technique which provides more reliable estimates of the entire spectrum, including negative Lyapuov exponents. The bad news is that the process is seriously degraded by even small amounts of noise. The last paper in this chapter, Parlitz' (1992) 'Identification of True and Spurious Lyapunov Exponents from Time Series' suggests trolling with time reversals which should reverse the sign of true Lyapunov exponents. The good news is that regularization and smoothing data contaminated with noise can overcome some of the degredative effects of noise.

The final chapter of this section contains four papers on *Periodic Orbits and Symbolic Dynamics*. These papers provide attractor reconstructions, return and Poincaré maps, and parameter maps from the analysis of various physical systems.

Part III (pp. 206-395) deals with the practical problems of "Prediction, Filtering, Control and Communication in Chaotic Systems." The editors note that after the initial development of the concept of reconstruction of the attractor (Taken's embedding theorem, 1981), estimating the correlation dimension (Grassberger & Procaccia, 1983), reconstruction of the local linear dynamics (Eckmann & Ruelle, 1985), and the approximation of Lyapunov exponents (Sano & Sawada, 1985), the way was paved for the development of ideas of prediction and nonlinear forecasting.

The first chapter (10), *Prediction*, contains four principal papers in this development: Farmer and Sidorowich (1987) who "discuss the scaling of prediction error for several artificial and experimental time series," Casdagli (1989) who "compares local linear approximations to the dynamics with global approximations, in the form of radial basis functions," Sugihara & May (1990) who use local linear short term techniques and discuss difficult issues of measurement error, and Sauer (1993) "using singlular value decomposition to restrict the dynamics to the tangent plane of the attractor, and using Fourier interpolation to counteract undersampling difficulties." Most of these use known systems to calibrate the techniques in addition to testing them on experimental data.

Chapter 11 on *Noise Reduction* is important because distinguishing noise from deterministic chaos is difficult due to the continuous nature of chaotic power spectra. Three papers by Kostgelich & Yorke (1988), Grassberger et al. (1993), and Hammel (1990) are included.

Chapter 12, *Control: Theory of Stabilization of Unstable Orbits* contains four papers which deal with the theory of stabilizing periodic orbits in order to control undesirable chaotic orbits which depend on them.

Chapter 13, *Control: Experimental Stabilization of Unstable Orbits* contains 6 papers which provide examples of such control in physical and physiological systems such as a gravitationally buckling magnetoelastic ribbon, thermally driven fluid convection, an electrical circuit, rabbit cardiac tissue, a laser system, and the Belousov-Zhabotinskii reaction. Garfinkel, Spano, Ditto, and Weiss (1992), state that chaotic systems are "highly susceptible to control, provided that the developing chaos can be analyzed in real time and that analysis is then used to make small control interventions. . . . By administering electrical stimuli to the heart at irregular times determined by chaos theory, the arrhythmia was converted to periodic beating." Besides physiological issues of health and disease, control theory could be considered for use in psychological therapy, and in the control of organizations and social systems. Can we borrow some of these techniques in our fields where noise becomes more prominant? Time will tell.

Chapter 14, *Control: Targeting and Goal Dynamics* contains three papers which go beyond the reduction of undesirable chaotic behavior and tries to restrict the orbit to "a small region about some specified point on the chaotic attractor" or to a specific dynamical goal.

Chapter 15, the final chapter, on *Synchronism and Communication*, at first blush would seem to be the ultimate in esoteric physical systems that, with their low noise conditions, might have the least applicability to psychological and social systems. In living systems they can be of great import, and do have potential applications of precision control. On the other hand, these topics involve the aspects of psychological and social systems, the self-organizational features, that we are most concerned with. Thus these techniques might help in dealing with communication and harmony within and between individuals or within and between components of social organization. Becoming more familiar with some of these technical considerations may yield some more explicit formulations to our metaphorical fancies as well as some suggestions for psychological, social, and biological research.

For any of us interested in doing or reading research on chaotic systems, this book should become a cornerstone of our personal libraries and not left for an occassional visit at our institutional libraries. The clarity and elegance of the exposition is exceptional; the importance of the reprints undeniable.

Frederick David Abraham
Blueberry Brain Institute & Chaos Cooperative
Waterbury Center, VT
http://www.pacweb.com/blueberry

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