# Networks, networks of networks, and so on

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# Why networks?

- Framework for interdisciplinary applications
- Common language:
  - Theoretical side: graph theory (mathematics)
  - Applied side: “real” networks
- Nodes – vertices
- Links – edges
- Graphs and social networks have been around for a long time

# Networks and complexity

- Topology becomes a fundamental property to understand dynamics of complex systems
- Dense connectivity: all to all
- Two extreme cases of sparse connectivity:
  - regular
  - random
- New statistical properties of “complex” networks 1998–1999

# How real are real networks?

- Real nodes and links
  - Train network
  - Internet
  - …….
- Inferred from data
  - Finance
  - Climate
  - Functional networks in the brain

# Complexity at all scales

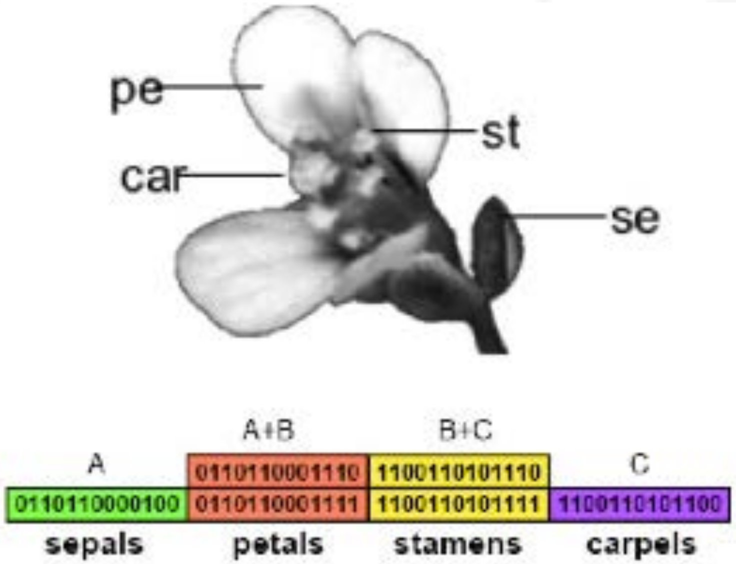

# Networks at all scales (of life)

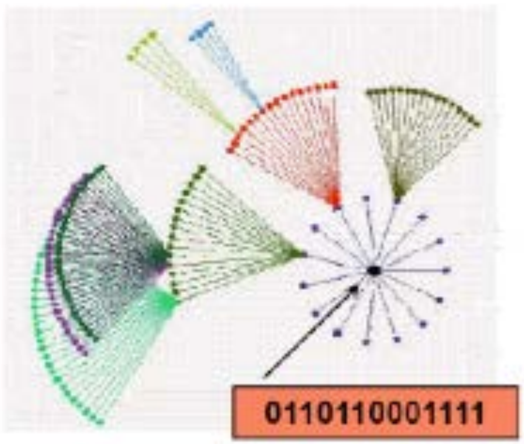
- Cell
- Brain
- Mind
- “real” contacts society
- “virtual” contacts society
- Internet

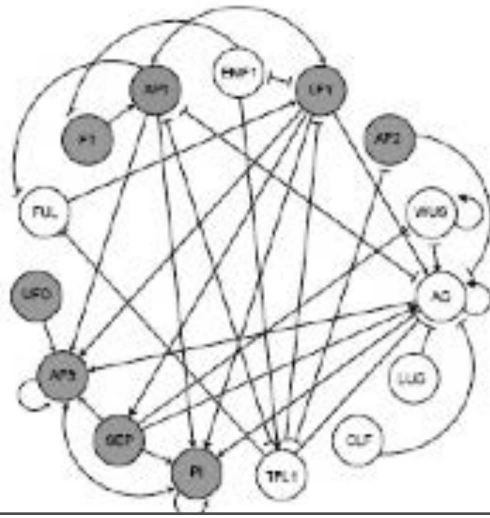
# Cellular networks

- Metabolic networks
- Protein interaction networks
- Gene regulatory networks

# Gene regulatory network



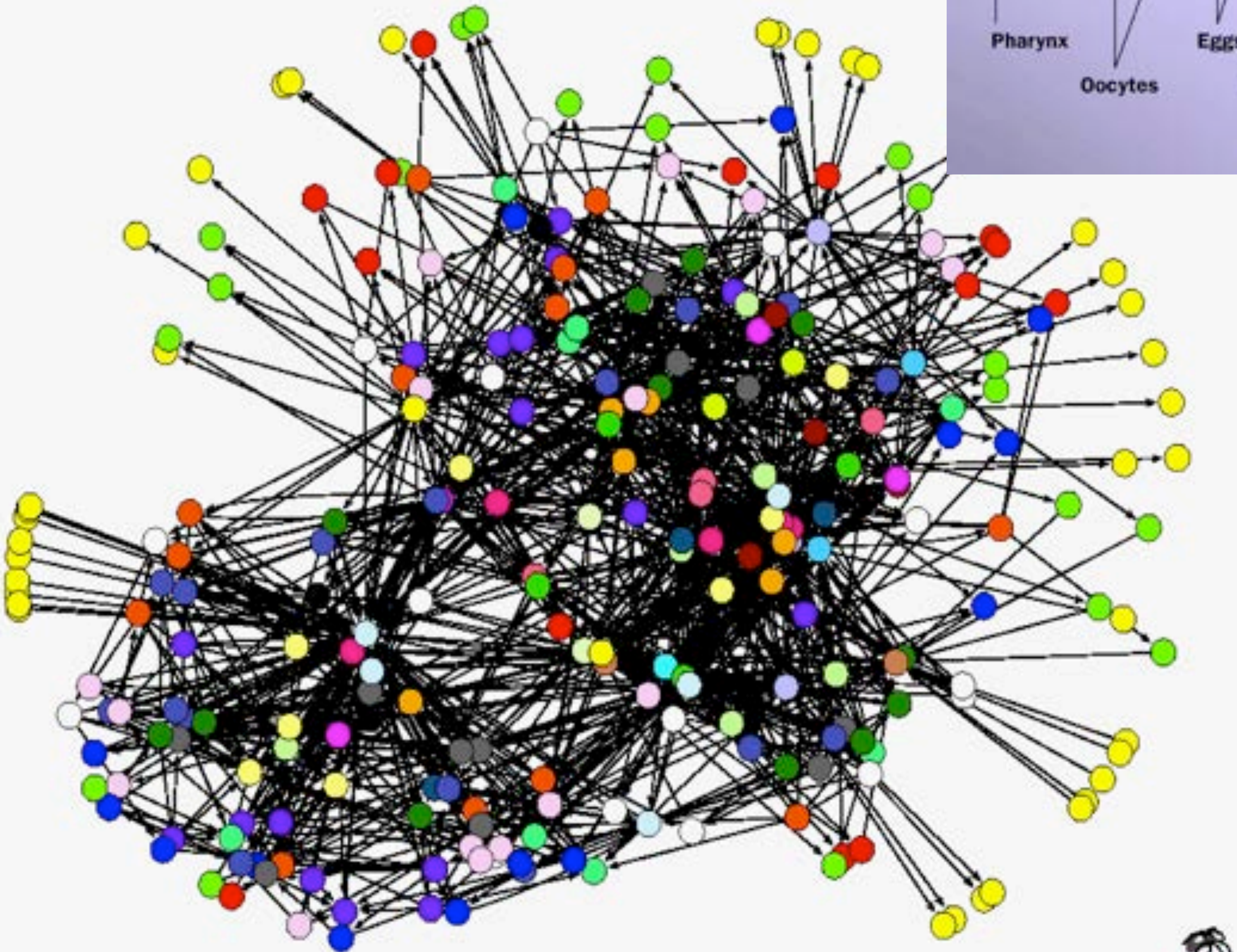



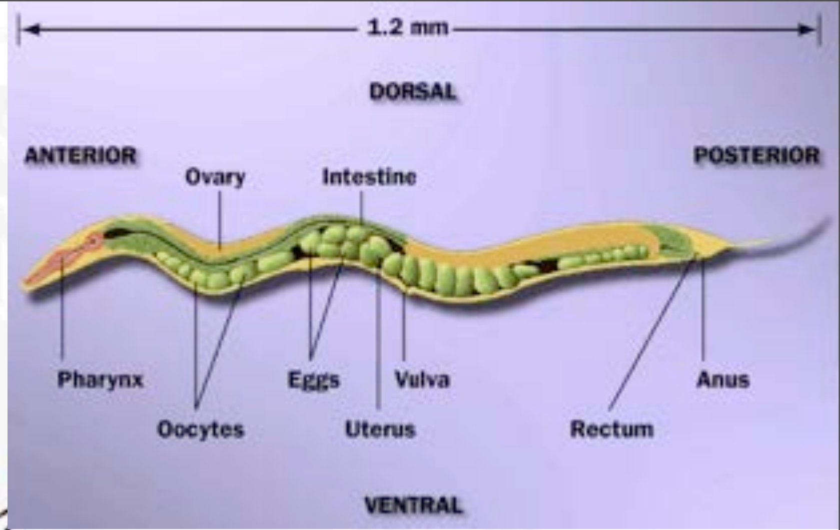



# Neural networks

- C. elegans
- Cultured neural networks

# C. elegans

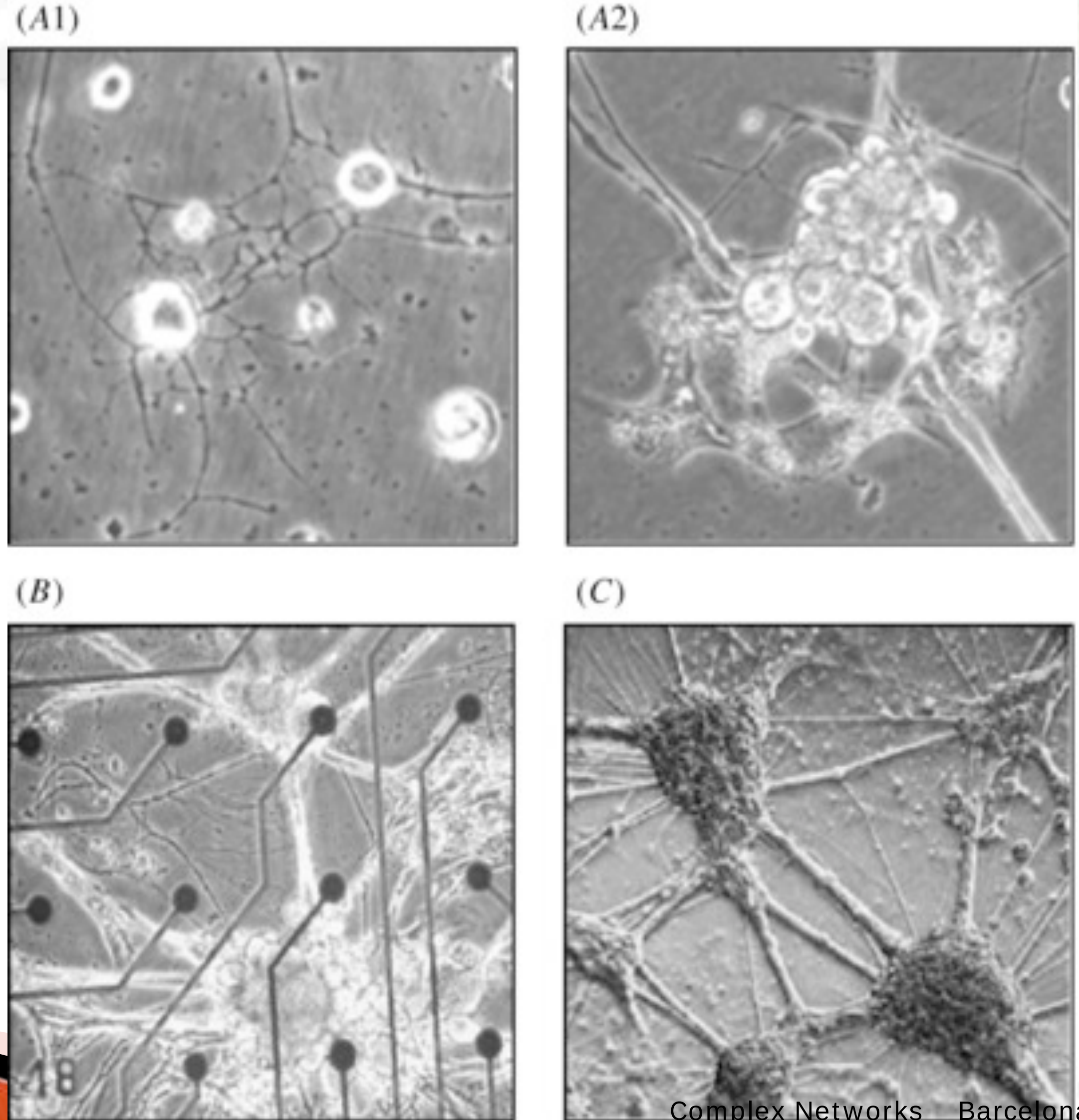



1.2 mm
DORSAL
ANTERIOR
POSTERIOR
Ovary
Intestine
Pharynx
Eggs
Vulva
Anus
Oocytes
Uterus
Rectum
VENTRAL

# Cultured networks

# Cultured networks

# Cultured networks



# Cortical networks

- Cortico-cortico interactions
  - Macaque
  - cat

# Cat cortex

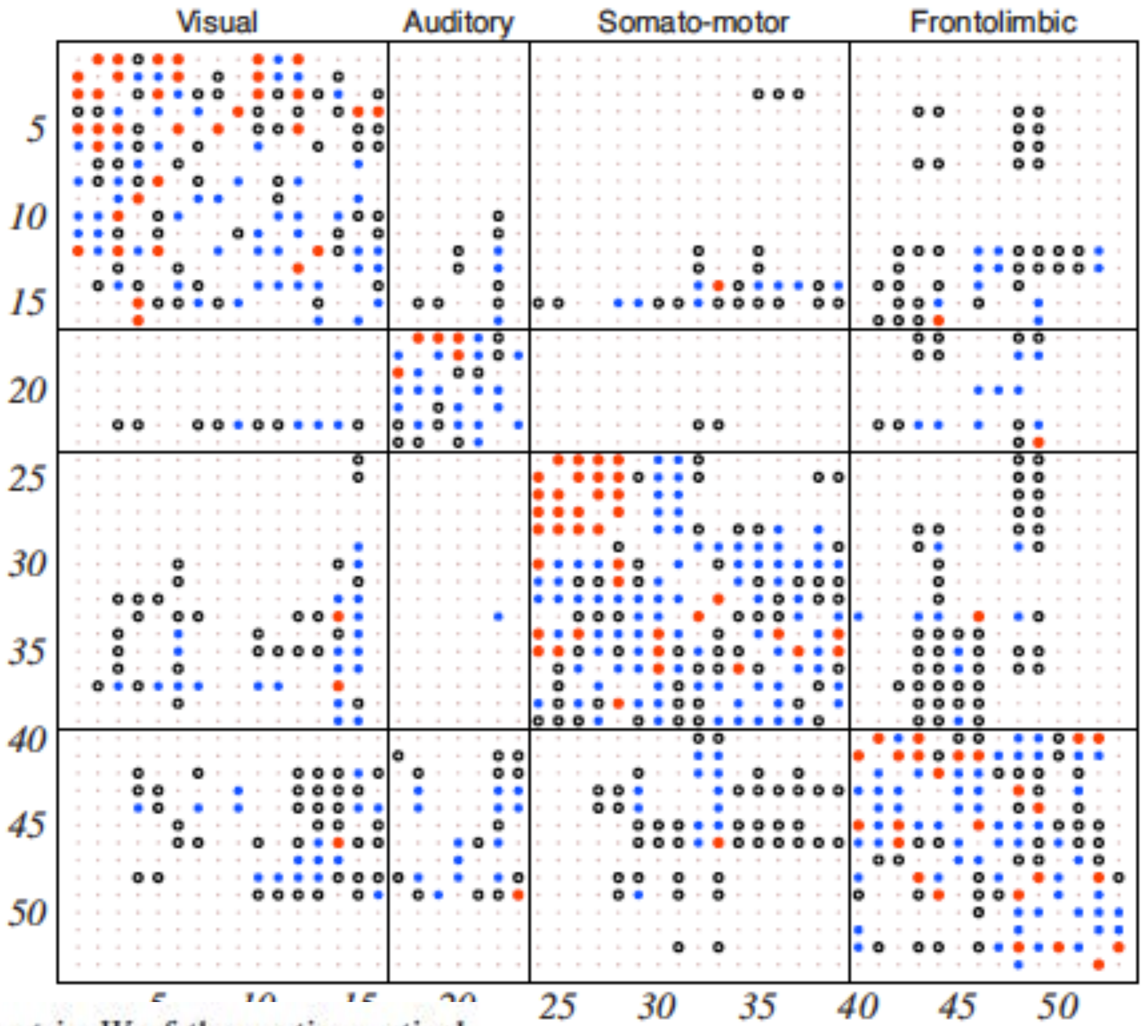



FIG. 1. (Color online) Weighted adjacency matrix $W$ of the corticocortical connectivity of the cat comprising of 826 directed connections between 53 cortical areas (Refs. 6 and 7). The connections are classified as weak (open circles), intermediate (blue stars), and dense (red filled circles) according to the axonal densities in the projections between two areas. For visualization purposes, the nonexisting connections (0) have been replaced by dots. The network has clustered organization, reflecting four functional subdivisions: visual, auditory, somatosensory motor, and frontolimbic.

# Brain networks

- Functional
  - fMRI
  - EEG
- Anatomical

# Functional brain networks

Article in Productio

International Journal of Bifurcation and Chaos, Vol. 20, No. 6 (2010) 1–10

DOI: 10.1142/S0218127410026757

## COMPLEX NETWORKS: NEW TRENDS FOR THE ANALYSIS OF BRAIN CONNECTIVITY

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# Functional brain networks

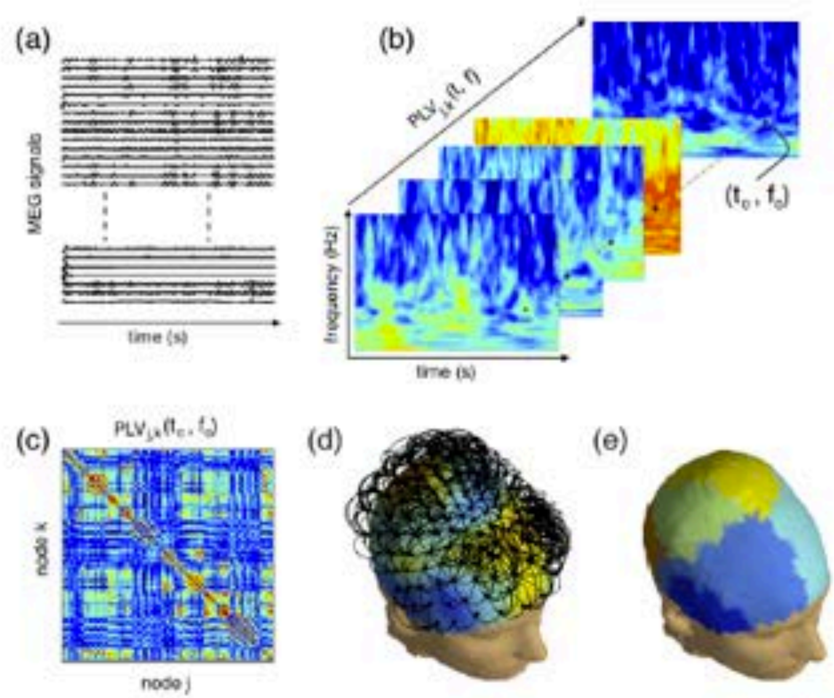


Figure 1: General scheme for the extraction of the time-varying brain networks: (a) signals are decomposed into time-frequency components to compute (b) pair-wise relations; (c) functional connectivity matrices are extracted at each point of the time-frequency space, defining (d) the functional brain networks used to extract the topological attributes (color codes the nodes degree) and the (e) modular structure (brain sites belonging to each module are arbitrarily colored). See details in the text.

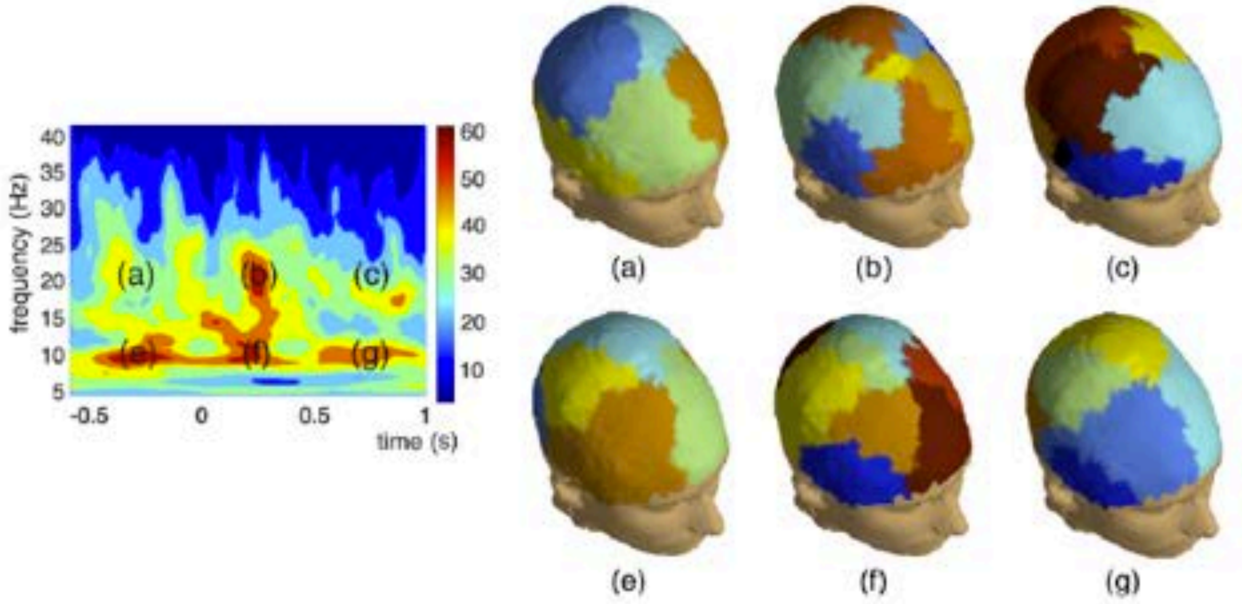


Figure 4: Topographical distribution of the modules extracted from brain networks at different time instants and frequencies: (a) time instant t = -0.25 s, frequency f =20 Hz; (b) t = 0.25 s after the presentation of the stimulus at f =20 Hz; (c) t =0.75 s, f =20 Hz; (e) t =-0.25 s, f =10 Hz; (f) t =0.25 s, f =10 Hz and (g) t =0.75 s, f =10 Hz. Brain sites belonging to each functional brain module were arbitrarily colored (there is no color correspondence between the modules of different networks). For the sake of clarity, isolated nodes were colored in black. (d) Time-frequency maps of mean degree is plotted to help network's localization in the time frequency space

# Cognitive level

- Semantic networks
- Association networks

# Semantic networks


**Cognitive Science: A Multidisciplinary Journal**
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t775653634

## The Large-Scale Structure of Semantic Networks: Statistical Analyses and a Model of Semantic Growth

Mark Steyvers [a]; Joshua B. Tenenbaum [b]
[a] Department of Cognitive Sciences, University of California, Irvine. [b] Department of Brain and Cognitive Sciences, Massachusetts Institute of Technology.

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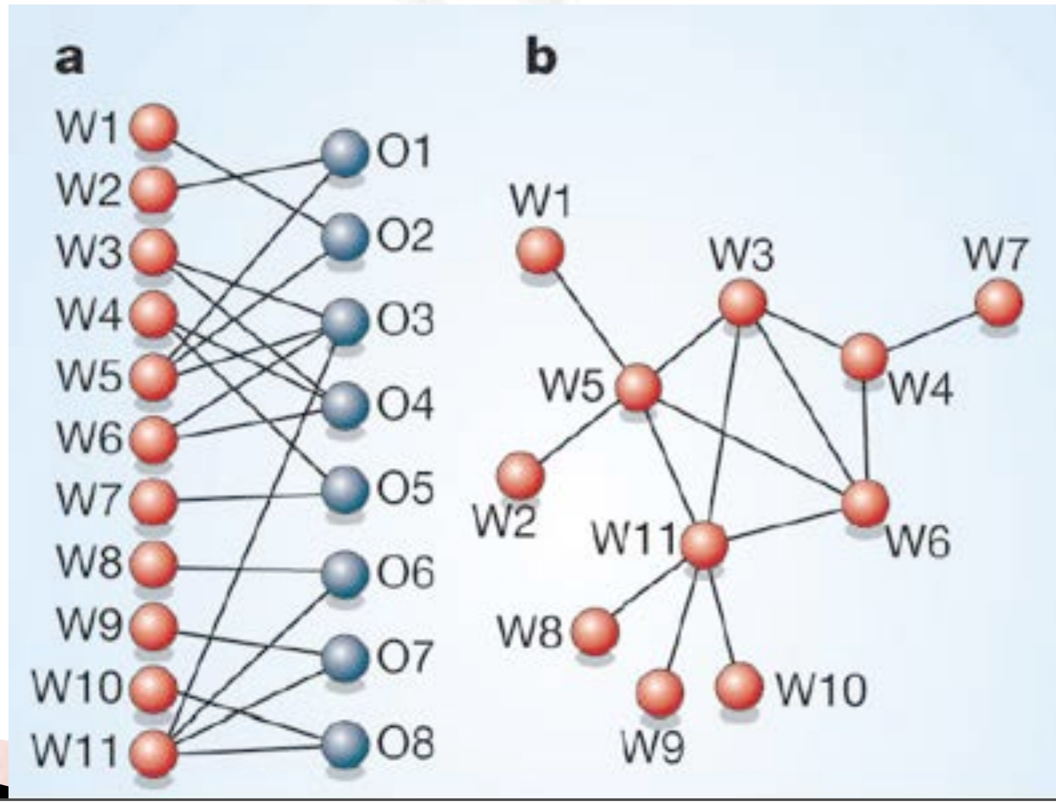
**URL:** http://dx.doi.org/10.1207/s15516709cog2901_3

# Associative network

- Words serve as cues (cat) for which participants have to write down the first word that comes to mind (dog)

# Theasaurus

- Words:
  - Semantic categories
  - Word forms and word meanings:
    - Synonymous
    - Polysemous



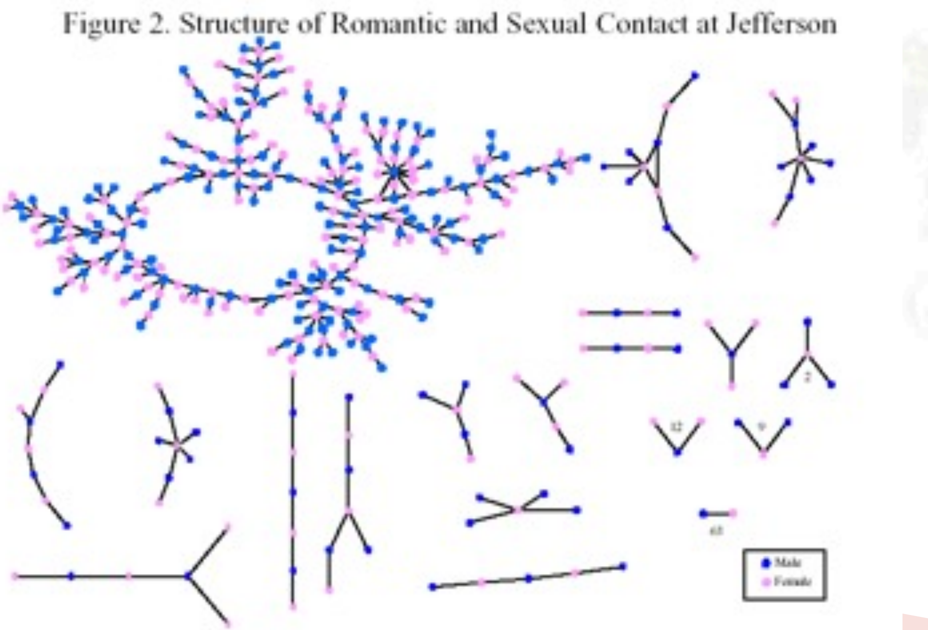
# Psycho-social networks

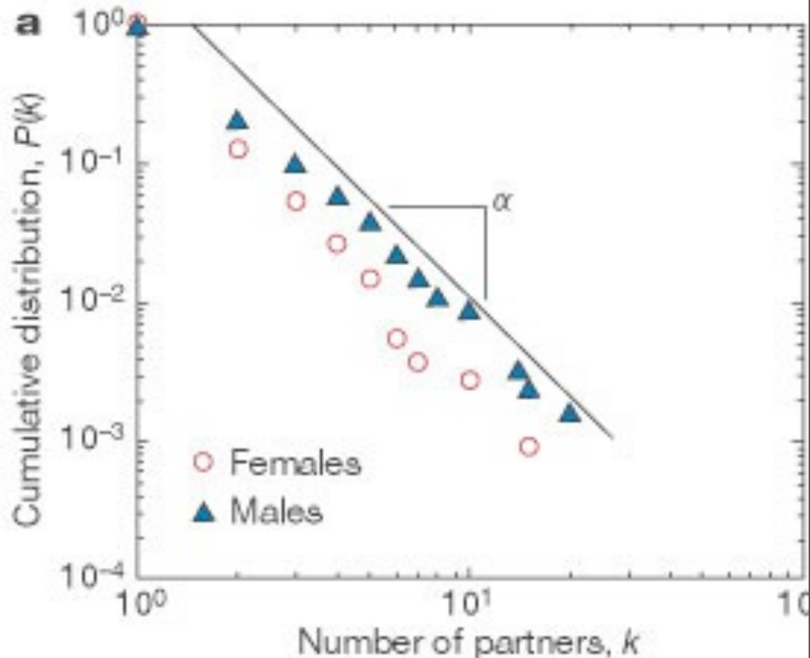
- Language networks: network derived for each language
  - http://www.linguistic-networks.net/
- Network of languages in the world

# Social networks: short

- Family
- Friendship
- Organizational
- Sexual contacts networks

# Sexual networks



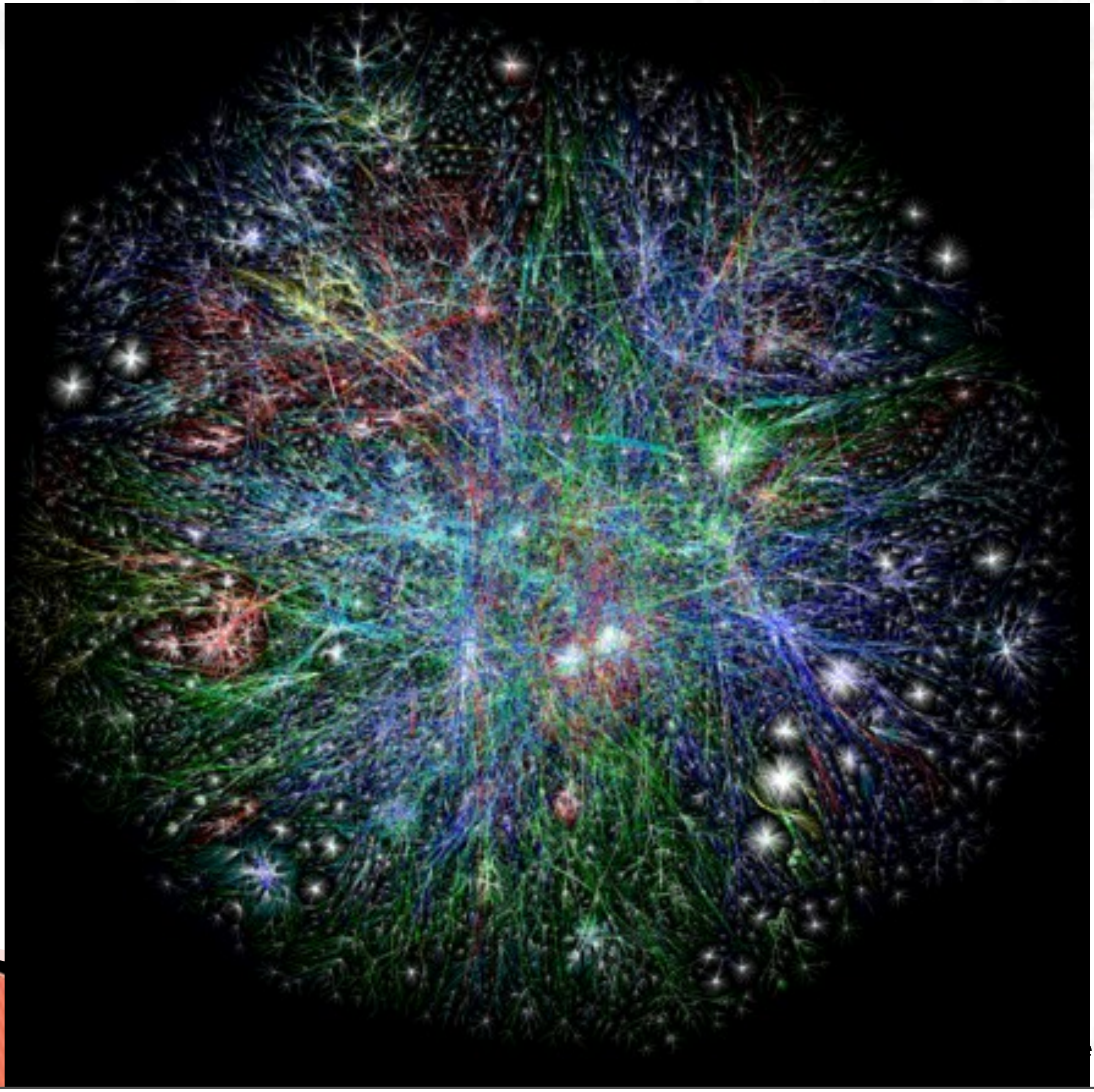
# Social networks: long

- Communications based
- Facebook
- Twitter
- Messaging
- Skype

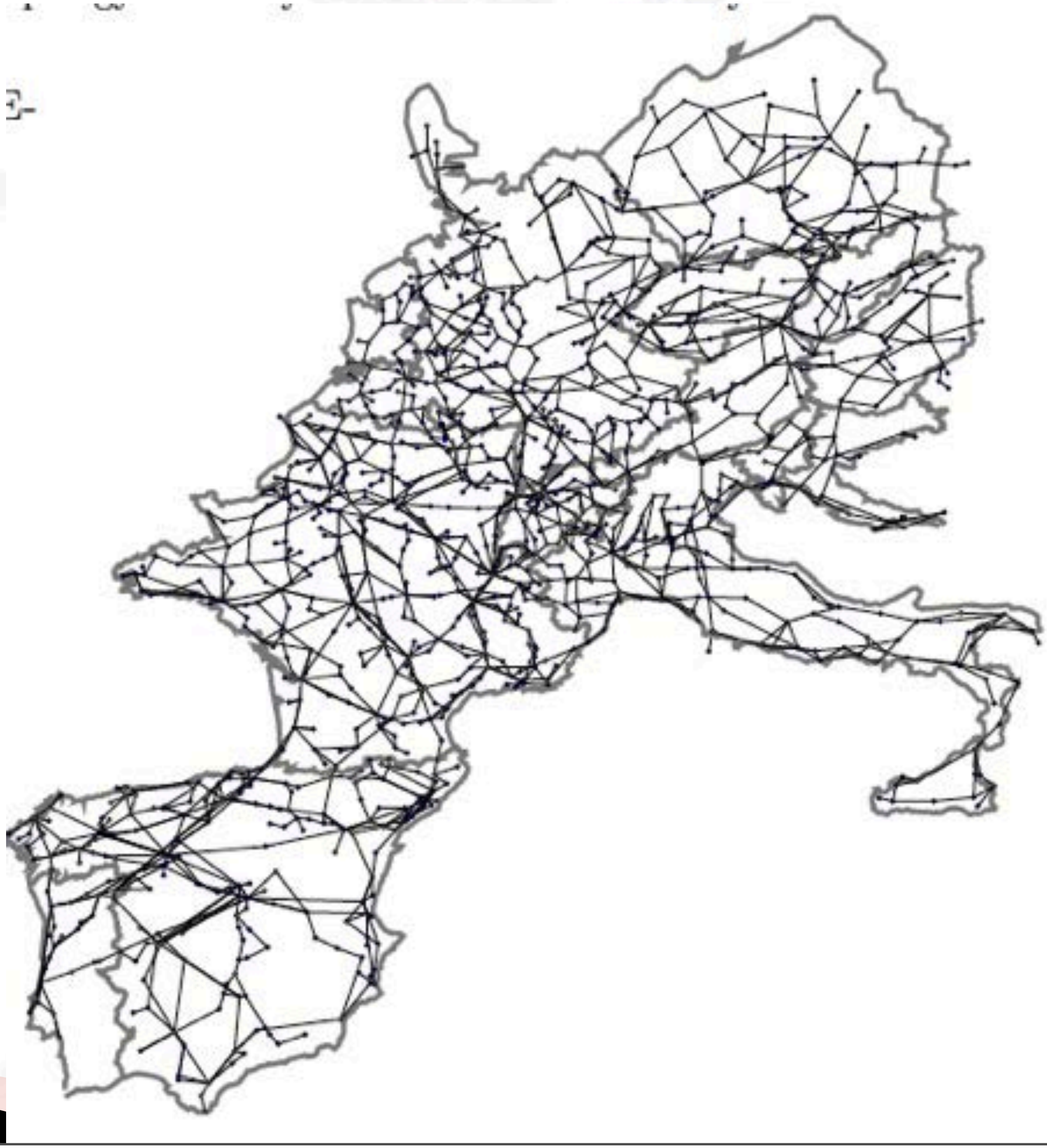
# Technological networks

- Internet
- Phone networks
- Transportation
- Power grid

# Internet



# Power grid



# New Complex Network Science

- Data mining: huge amounts of data
- Increase in computing power
- Statistical characterization
- Dynamics and topology
- Modelization
- Important contribution from Statistical Physics Community
- New Journals “Network Science” from CUP, ”Complex Networks” from OUP

# New challenges

- Networks of networks
- Multiplex networks
- Dynamical networks

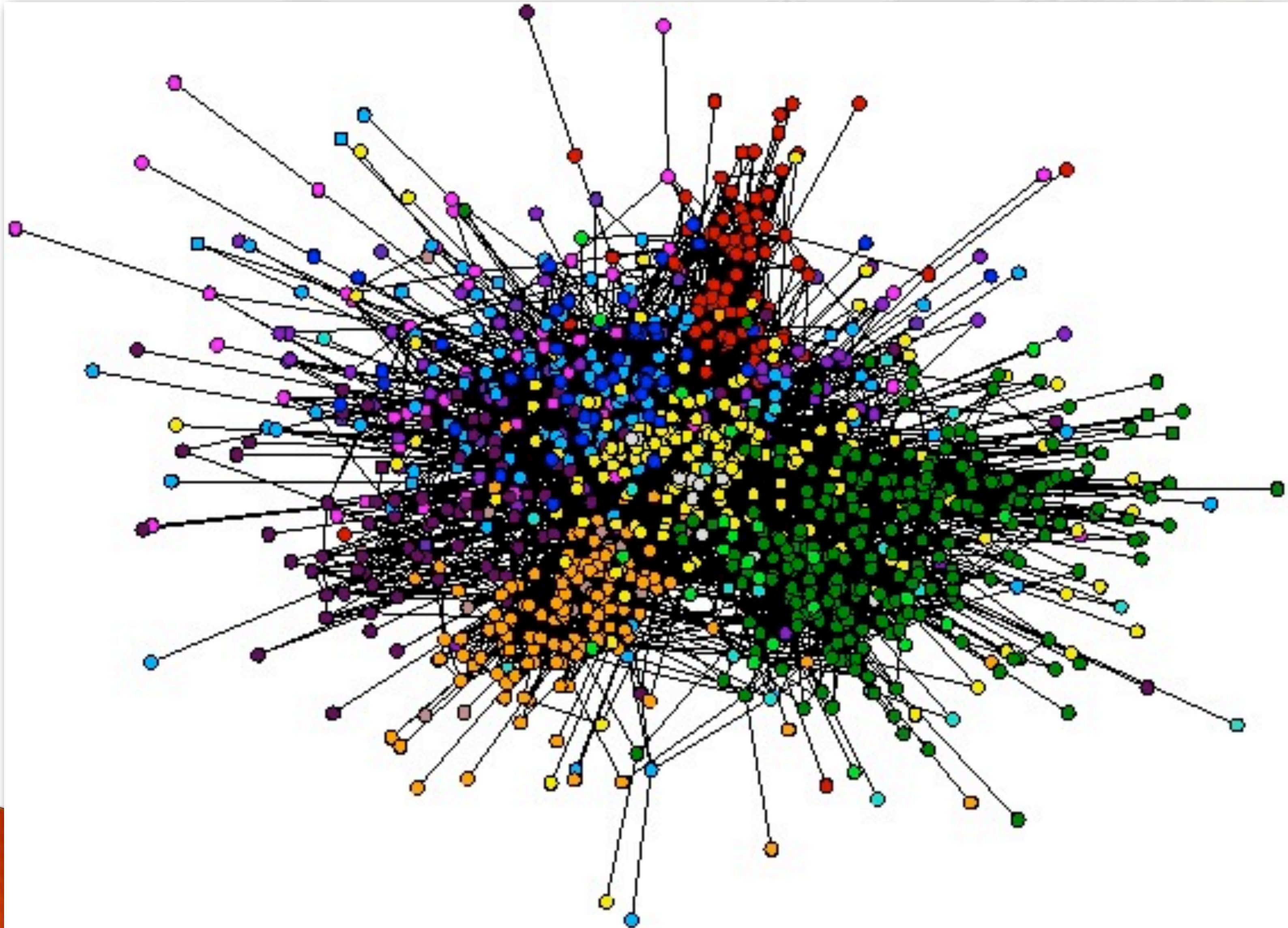
# Networks of networks

- Network are intertwined at different levels
- Cells interact with other cell: the genetic network inside the cell can produce some proteins that diffuse across cells affecting other cells. Diffusion is produced in the spatial lattice, nothing to do with the genetic network inside the cell.
- Cell are nodes and transportation networks for proteins

# Cultured networks

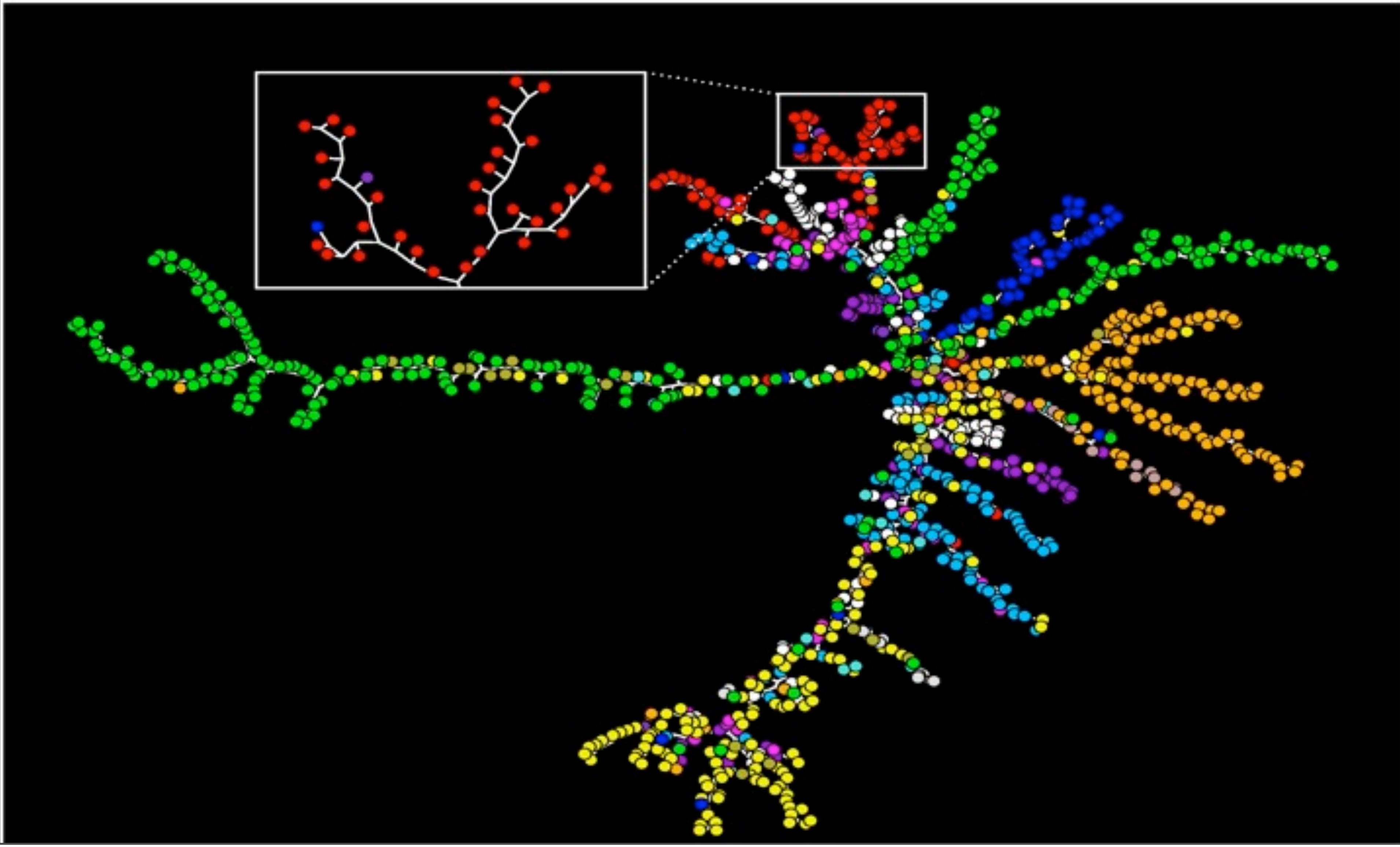
- Axons are the connections between neurons
- Long or short range
- But they can be clustered forming modules for some reason

# E-mail network

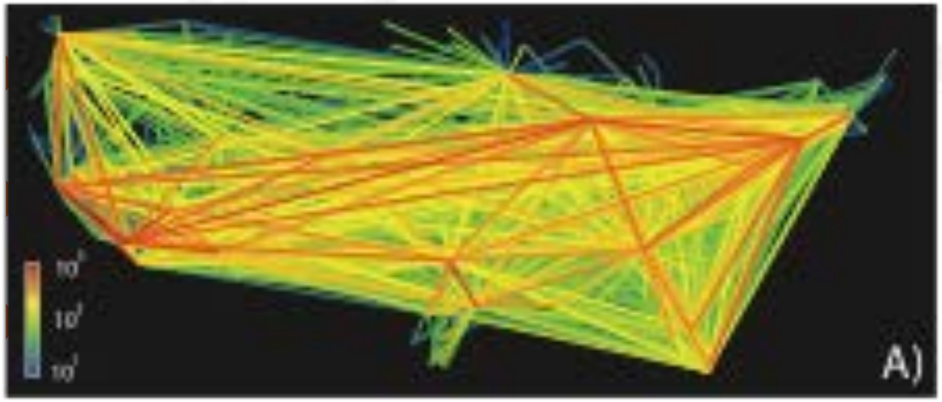


# Communities

# Communities



# Transportation




Airline transportation networks


Commuting network

Balcan et al., PNAS (2009)

# Dynamics in networks of networks

- Synchronization in the cat cortex
- Synchronization in hierarchically organized networks

# Why synchronization

- Neural activity in the brain
- Is synchronization good? NO
- Partial synchronization is not so bad!
- Synchrony and correlations in different areas of the brain at different levels of description

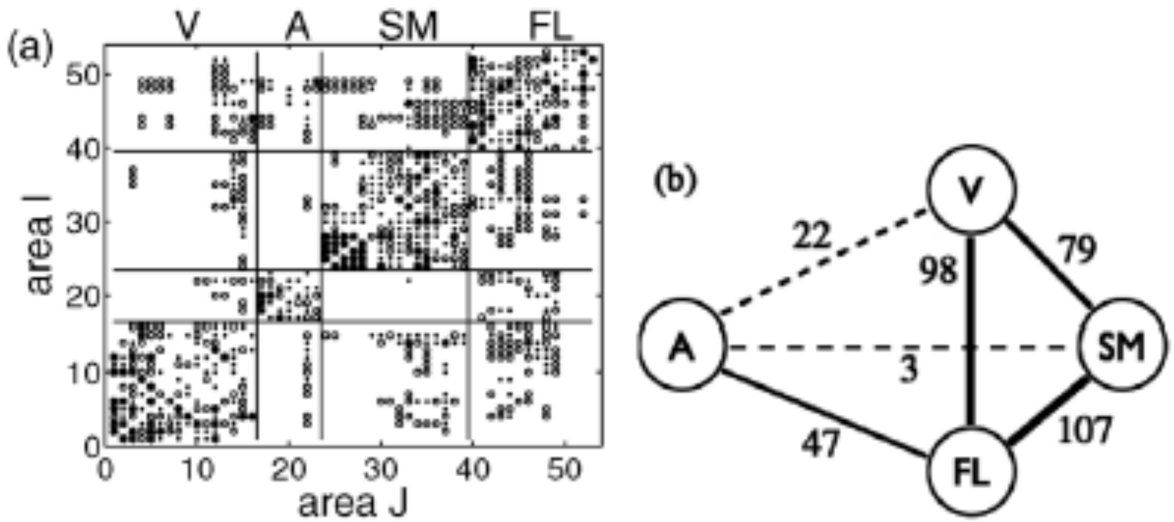
A. Arenas, A. D.-G., C. J. Pérez-Vicente, Phys. Rev. Lett (2006)
A. Arenas,A. D.-G., J. Kurths, Y. Moreno, C. Zhou, Phys. Rep (2008)

# Hierarchical Organization Unveiled by Functional Connectivity in Complex Brain Networks

Changsong Zhou,[1] Lucia Zemanová,[1] Gorka Zamora,[1] Claus C. Hilgetag,[2] and Jürgen Kurths[1]

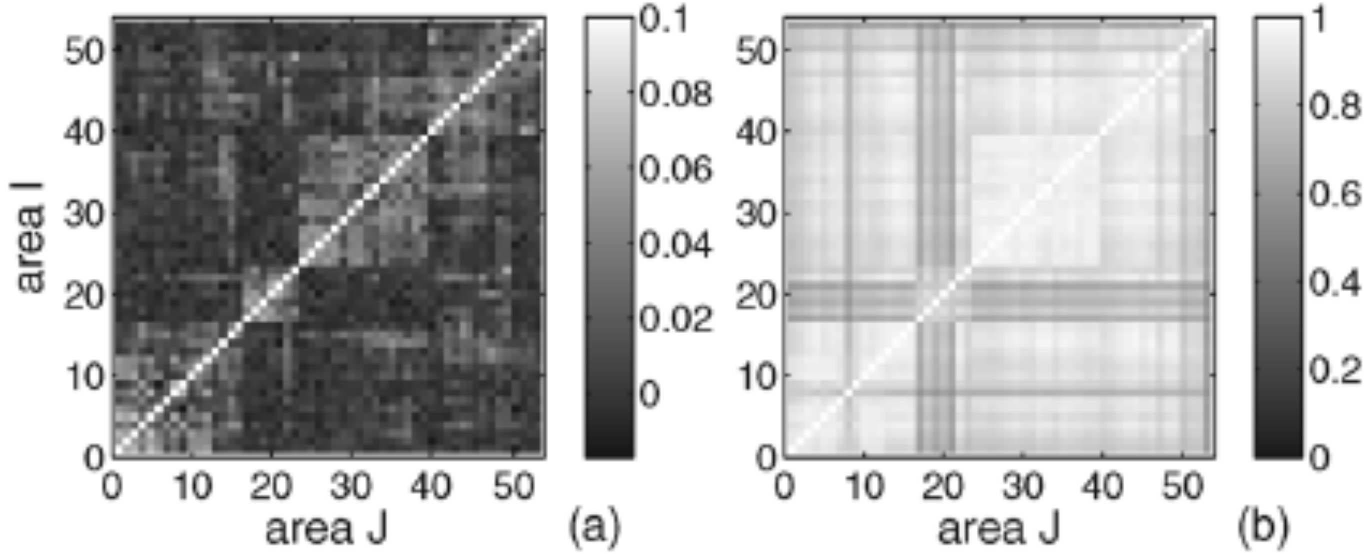
- 53 areas classified into
- 826 fibers of axons

four functional subdivisions (visual, auditory, somatosensory motor, and frontolimbic) displayed in Fig. 1. From the 826 connections, 470 are internal, i.e., they connect two areas in the same cluster, and 356 are external, i.e., connect two areas



# Network of networks in the model

- Each cortical area is modeled with a small-world of 200 neurons
- SW: local + random long-range shortcuts
- FitzHugh-Nagumo



# Synchronization dynamics

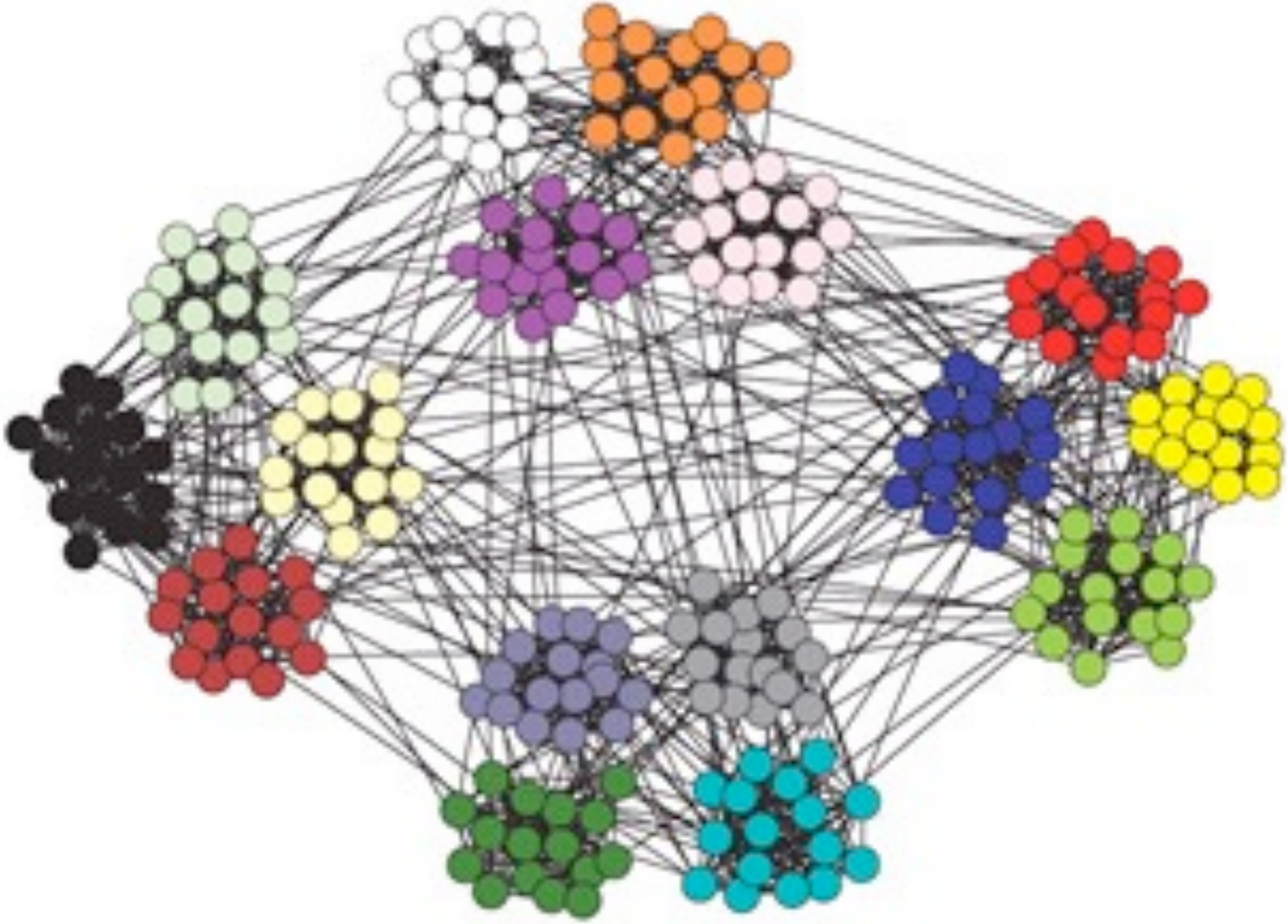
- Synchronization of Kuramoto oscillators

$$\frac{d\theta_i}{dt} = \omega_i + \sigma \sum_j A_{ij} \sin(\theta_j - \theta_i) \qquad i = 1, ..., N$$

Kuramoto (the Applet)

$$\frac{d\theta_i}{dt} = \sigma \sum_j A_{ij} \sin(\theta_j - \theta_i) \qquad i = 1, ..., N$$

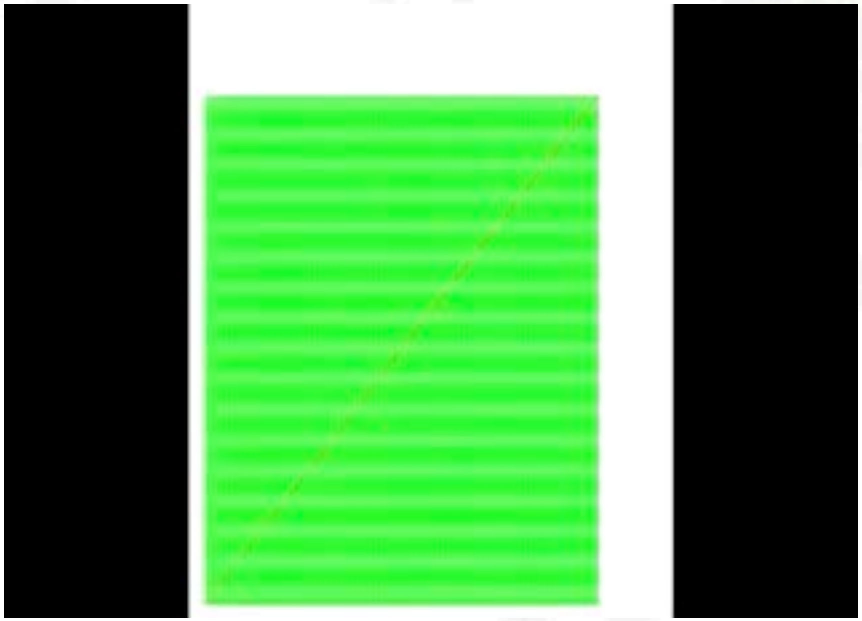
# Hierarchical structure of



# Time evolution of the correlation

$$\rho_{ij}(t) =< cos(\theta_i(t) - \theta_j(t)) >$$

# Time evolution of the correlation



$$\rho_{ij}(t) = < cos(\theta_i(t) - \theta_j(t)) >$$

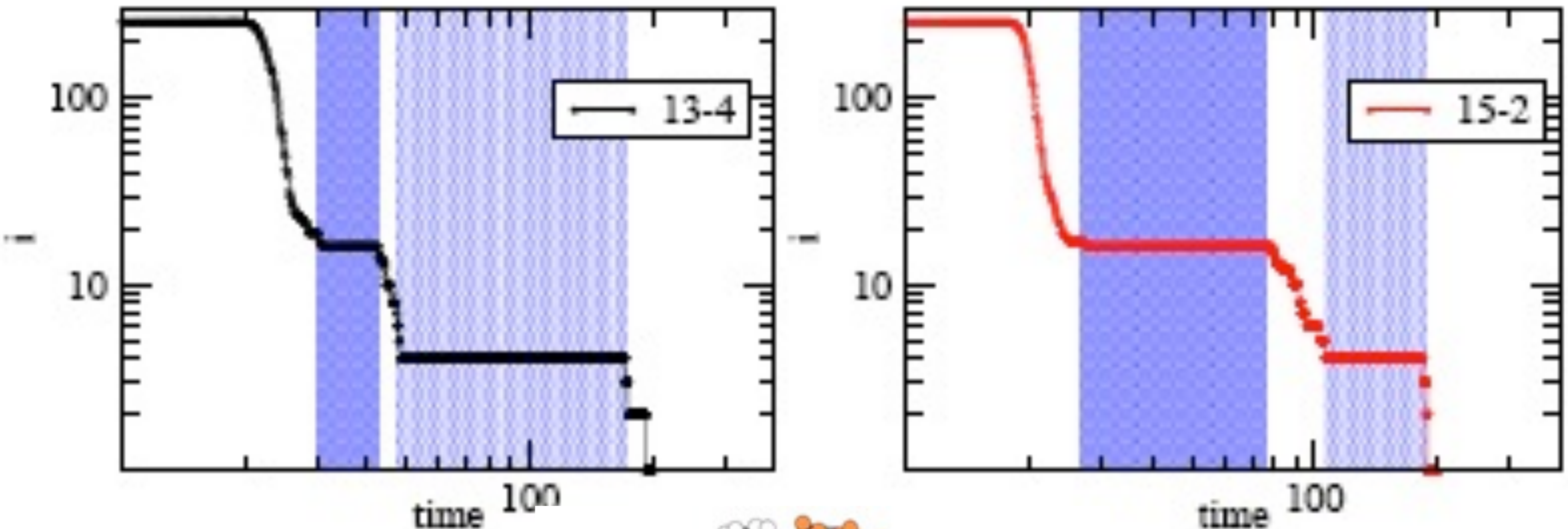
# New graphical representations

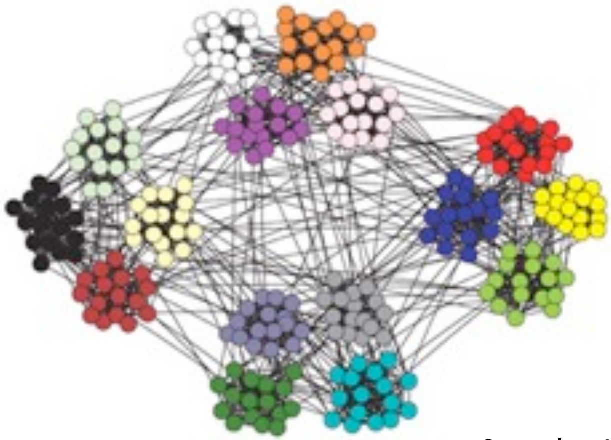
- Two nodes are connected if they are "synchronized"
- Dynamic connectivity matrix

$$\mathcal{D}_t(T)_{ij} = \begin{cases} 1 & \text{if } \rho_{ij}(t) > T \\ 0 & \text{if } \rho_{ij}(t) < T \end{cases}$$

- Fixed time – moving threshold
- Fixed threshold – time evolution of the network
- Structure at different time scales

# Number of connected components



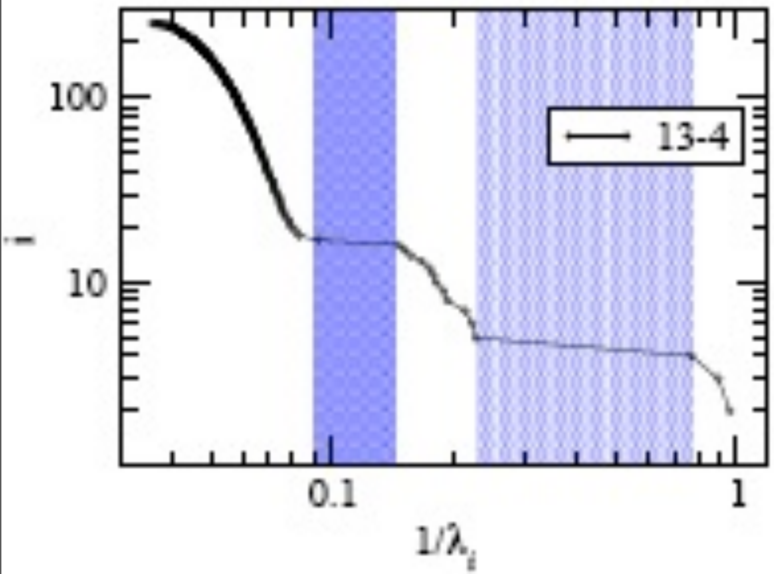



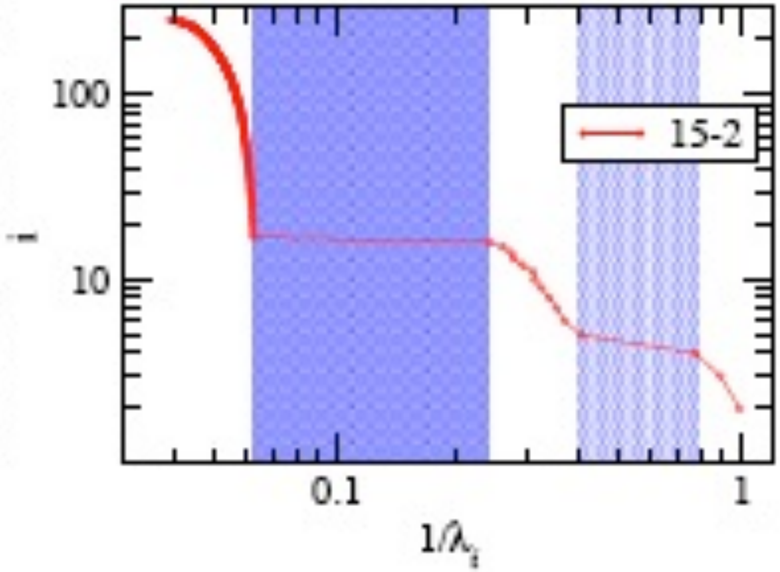
# Spectral properties

- Spectrum of the Laplacian matrix
- We order the eigenvalues

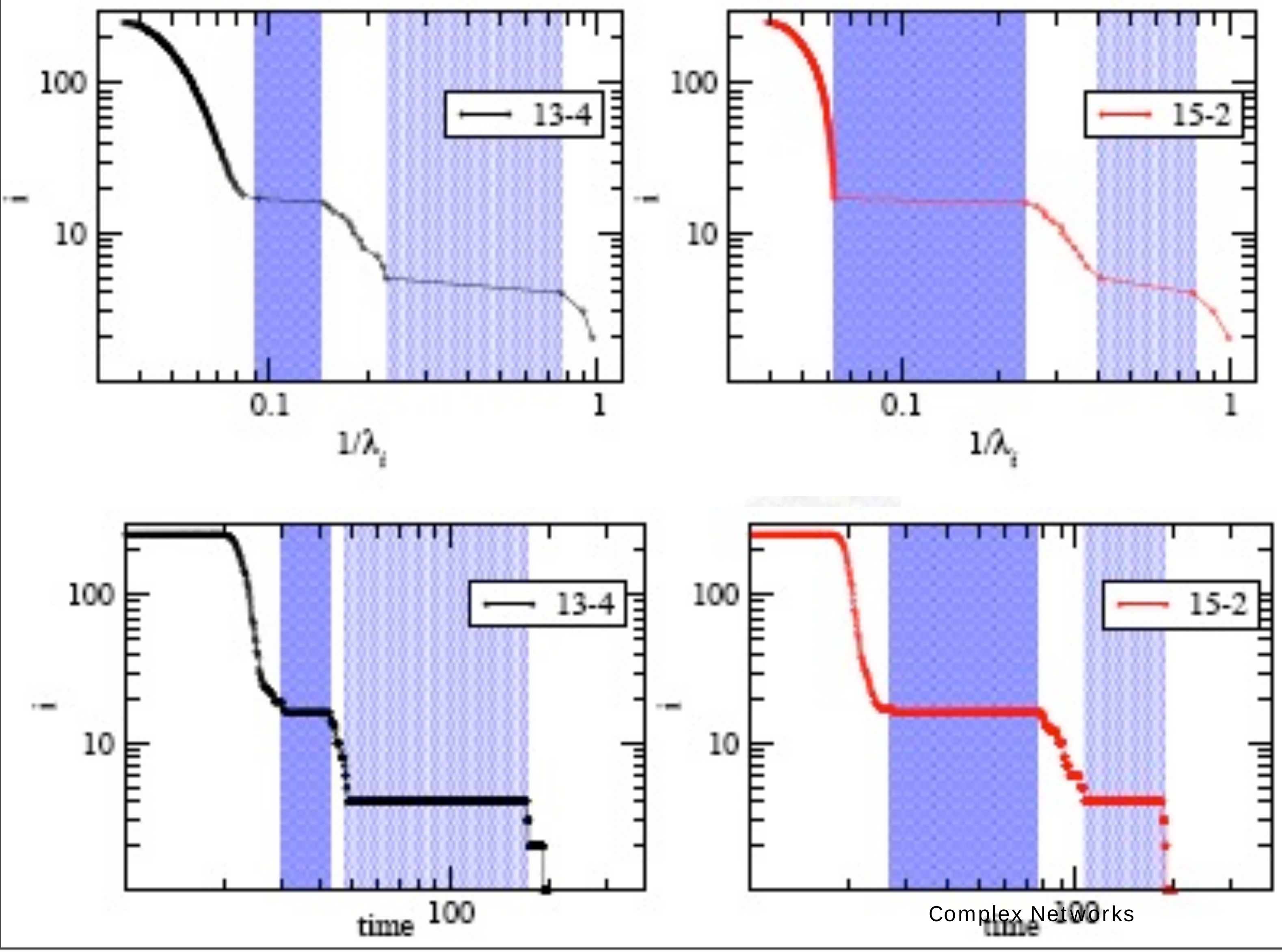
$$0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$$

# Spectral versus dynamics



# Spectral versus dynamics



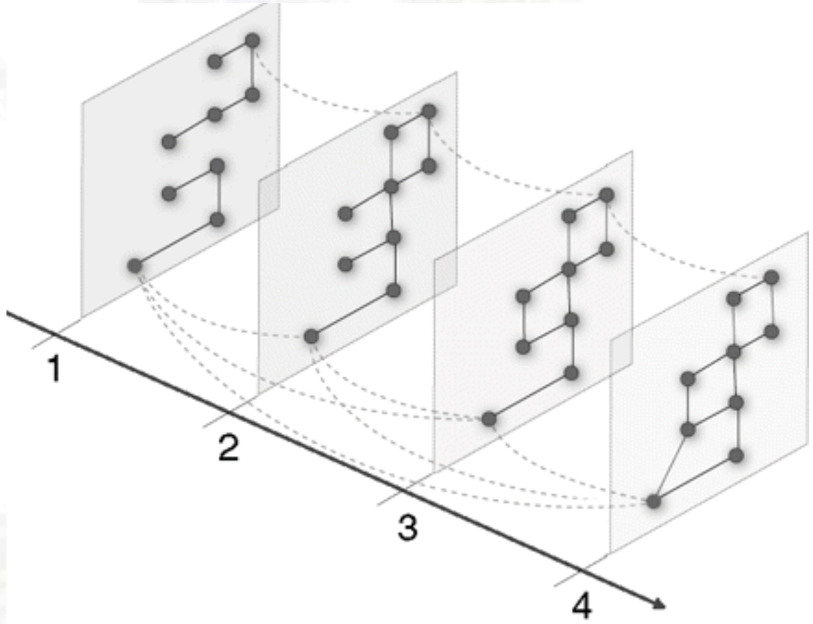
# Dynamics: useful to recover

- Degree of a node
- Modules structure

L. Prignano, A.D.-G. Phy. Rev. E (in press)
L. Prignano, Y. Moreno, A.D-G. Phys. Rev. E (submitted)

# Multiplex networks

- Networks defined (simultaneously) at different layers that interact
- Examples:
  - communicating: email, twitter, facebook, messaging
  - moving in a city:pedestrian paths, public transportation,"bicing", taxis,



# Time dependent networks

- Dynamics is not only on the nodes, it can also be on the links (and on both)

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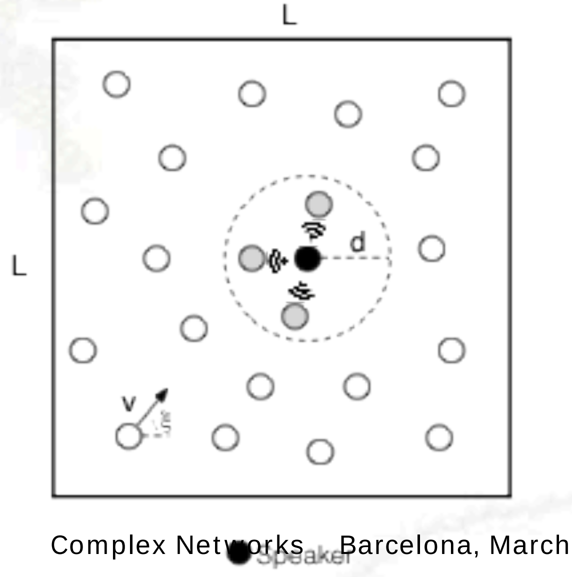
- Dynamics is not only on the nodes, it can also be on the links (and on both)

# Contact networks: short range

N. Fujiwara, J. Kurths, A. D.-G. Phys Rev E (2011)
A. Baronchelli, A. D.-G. Phys Rev E (2012)
L. Prignano, O. Sagarra, P.M. Gleiser, A. D.-G. Int. J. Bif. Chaos (in press)



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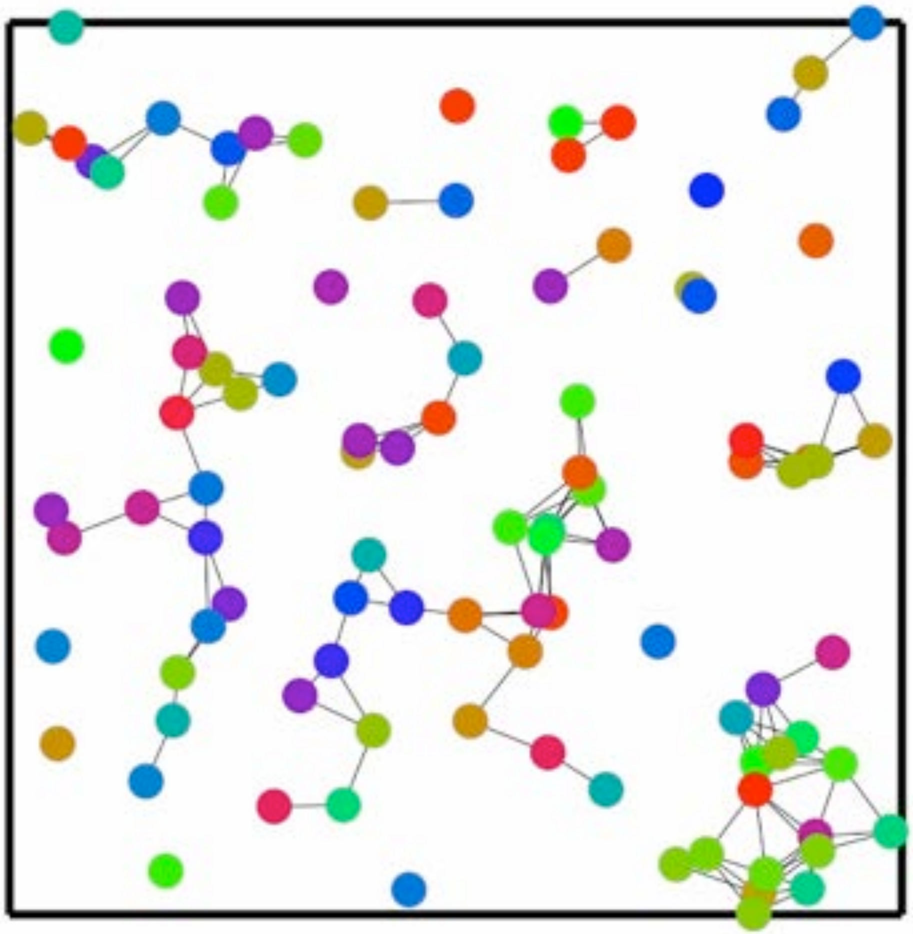

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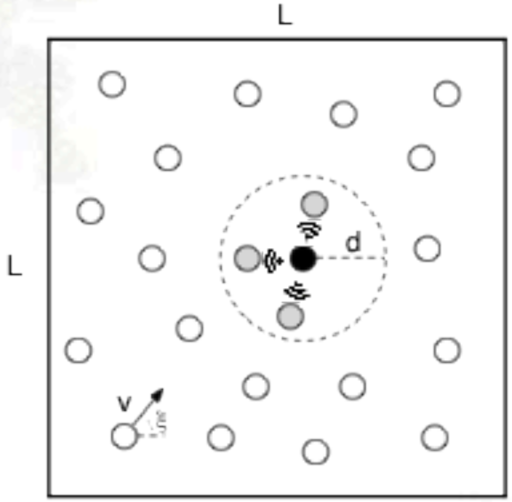

# New tools

- Time dependent spectral properties
- Expansions for different layers

# Conclusions

- Complexity at all scales
- Networks at all scales, interacting to give rise to networks at higher scales
- Are complex networks complex enough?
- Let's add some additional complexity:
  - Networks of networks
  - Multiplexity
  - Time dependent